

## Cts Time Markov Chain Homework

1. Do number 6.16 from Ross.
2. A salesman flies around between Atlanta, Boston, and Chicago as follows. She stays in each city for an exponential amount of time with mean  $1/4$  if the city is A or B, but with mean  $1/5$  month if the city is C. From A she goes to B or C with probability  $1/2$  each; from B she goes to A with probability  $3/4$  and to C with probability  $1/4$ ; from C she always goes back to A. What is the limiting fraction of time she spends in each city? Do this problem by hand—not with a computer.
3. Consider two machines that are maintained by a single repairman. Machine  $i$  functions for an exponentially distributed amount of time with rate  $\lambda_i$  before it fails. The repair times for each unit are exponential with rate  $\mu_i$ . They are repaired in the order in which they fail.
  - (a) Let  $X(t)$  be the number of working machines at time  $t$ . Is  $X(t)$  a Markov chain?
  - (b) Formulate a Markov chain model for this situation with state space  $\{0, 1, 2, 12, 21\}$ . In other words give the rate matrix,  $Q$ .
  - (c) Suppose that  $\lambda_1 = 1, \mu_1 = 2, \lambda_2, \mu_2 = 4$ . Use R to find the stationary distribution.
4. A job shop consists of three machines and two repairmen. The amount of time a machine works before breaking down is exponentially distributed with mean 10. If the amount of time it takes a single repairman to fix a machine is exponentially distributed with mean 8, then
  - (a) What is the average number of machines not in use?
  - (b) What proportion of time are both repairmen busy?