

Cts Time Markov Chain Homework

1. Do number 6.16 from Ross.
2. A salesman flies around between Atlanta, Boston, and Chicago as follows. She stays in each city for an exponential amount of time with mean $1/4$ if the city is A or B, but with mean $1/5$ month if the city is C. From A she goes to B or C with probability $1/2$ each; from B she goes to A with probability $3/4$ and to C with probability $1/4$; from C she always goes back to A. What is the limiting fraction of time she spends in each city? Do this problem by hand—not with a computer.
3. Consider two machines that are maintained by a single repairman. Machine i functions for an exponentially distributed amount of time with rate λ_i before it fails. The repair times for each unit are exponential with rate μ_i . They are repaired in the order in which they fail.
 - (a) Let $X(t)$ be the number of working machines at time t . Is $X(t)$ a Markov chain?
 - (b) Formulate a Markov chain model for this situation with state space $\{0, 1, 2, 12, 21\}$. In other words give the rate matrix, Q .
 - (c) Suppose that $\lambda_1 = 1, \mu_1 = 2, \lambda_2, \mu_2 = 4$. Use R to find the stationary distribution.
4. A job shop consists of three machines and two repairmen. The amount of time a machine works before breaking down is exponentially distributed with mean 10. If the amount of time it takes a single repairman to fix a machine is exponentially distributed with mean 8, then
 - (a) What is the average number of machines not in use?
 - (b) What proportion of time are both repairmen busy?