

Statistics 463
First Midterm Exam
October 5, 2006

Name: _____

Make sure to put all your answers in the space provided, and do not write on the plots. Also, be sure to give complete answers and to show your work.

1. (15 pts) Show that the following time series is stationary

$$x_t = w_t w_{t-2} + c$$

where c is a constant and w_t is a Gaussian white noise process with variance σ^2 .

2. (15 pts) Reminding you that an AR(1) process (with $|\phi_1| < 1$) may be written as

$$x_t = \sum_{j=0}^{\infty} \phi_1^j w_{t-j}$$

and that a geometric series in general can be written as

$$\frac{1}{1-y} = \sum_{j=0}^{\infty} y^j, \quad |y| < 1$$

find the mean and variance of x_t .

3. (35 pts) The data shown in figure 1 are monthly series of unemployed women between 16 and 19 from January 1961 to August 2002 and the differenced version of this data.

(a) Using ONLY the figure 1, does `employ` appear to be stationary data? How about the differenced version of the data?

(b) Given the appearance of the differenced data, do you think there is a trend in the original data? Do you think it would be a good idea to smooth the original data?

(c) Interpret and comment on the ACF plots in figure 2.

4. (35 pts)

(a) In the monthly data in figure 3, there is a moving average estimate of the mean with a window size of 12 and a moving average estimate with a window size of 3. Why is there such a large difference between the two. Could there be situations when both estimates could be useful?

(b) Discuss the plots of the residuals found in figure 4. Are the residuals stationary in both cases?

(c) Interpret and comment on the ACF plots of the residuals found in figure 5. Can both plots be interpreted as the dependency of the time series at certain lags?

(d) Figure 6 shows the periodogram of the residuals after the moving average estimate of the mean with window size 12 has been removed. Interpret the plot. Is this plot surprising given the plot of the actual residuals and the original data?

Figure 1: The top plot is the monthly time series of unemployed women between 16 and 19 from January 1961 to August 2002. The lower plot is a differenced version of this data.

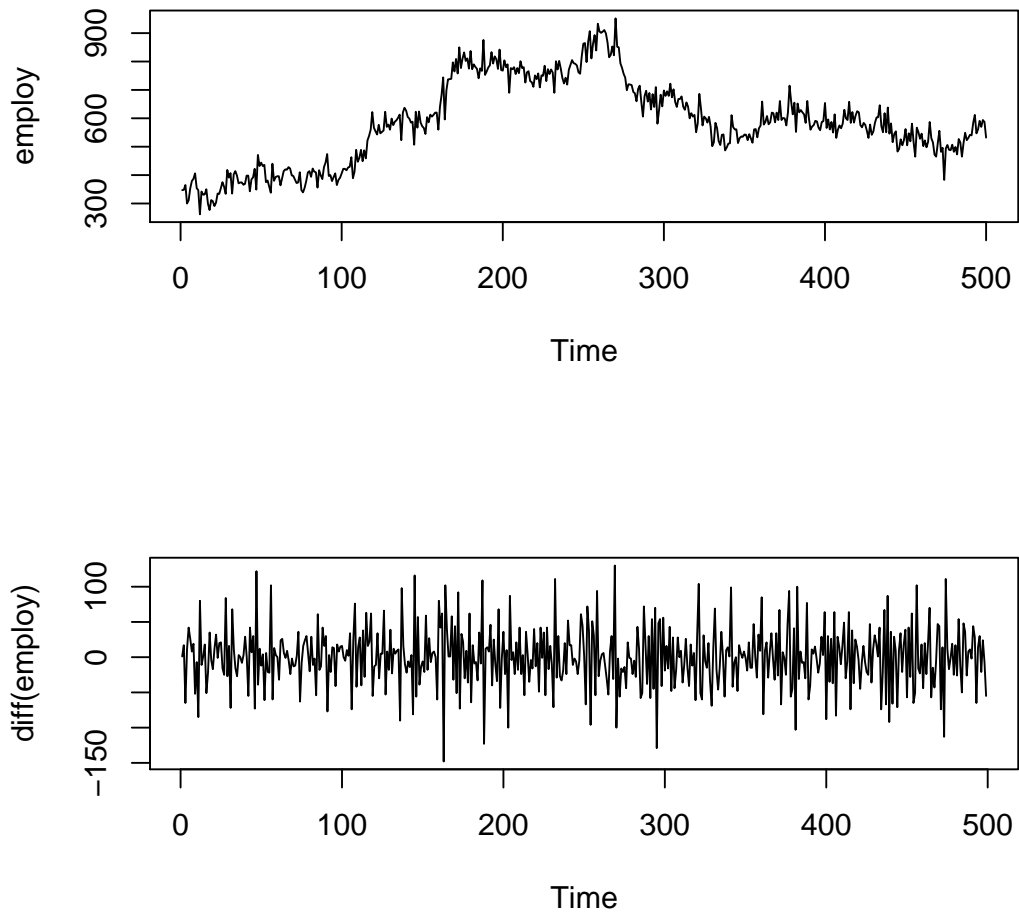


Figure 2: The top plot is the acf for the monthly time series of unemployed women between 16 and 19 from January 1961 to August 2002. The lower plot is an ACF plot of the differenced version of this data.

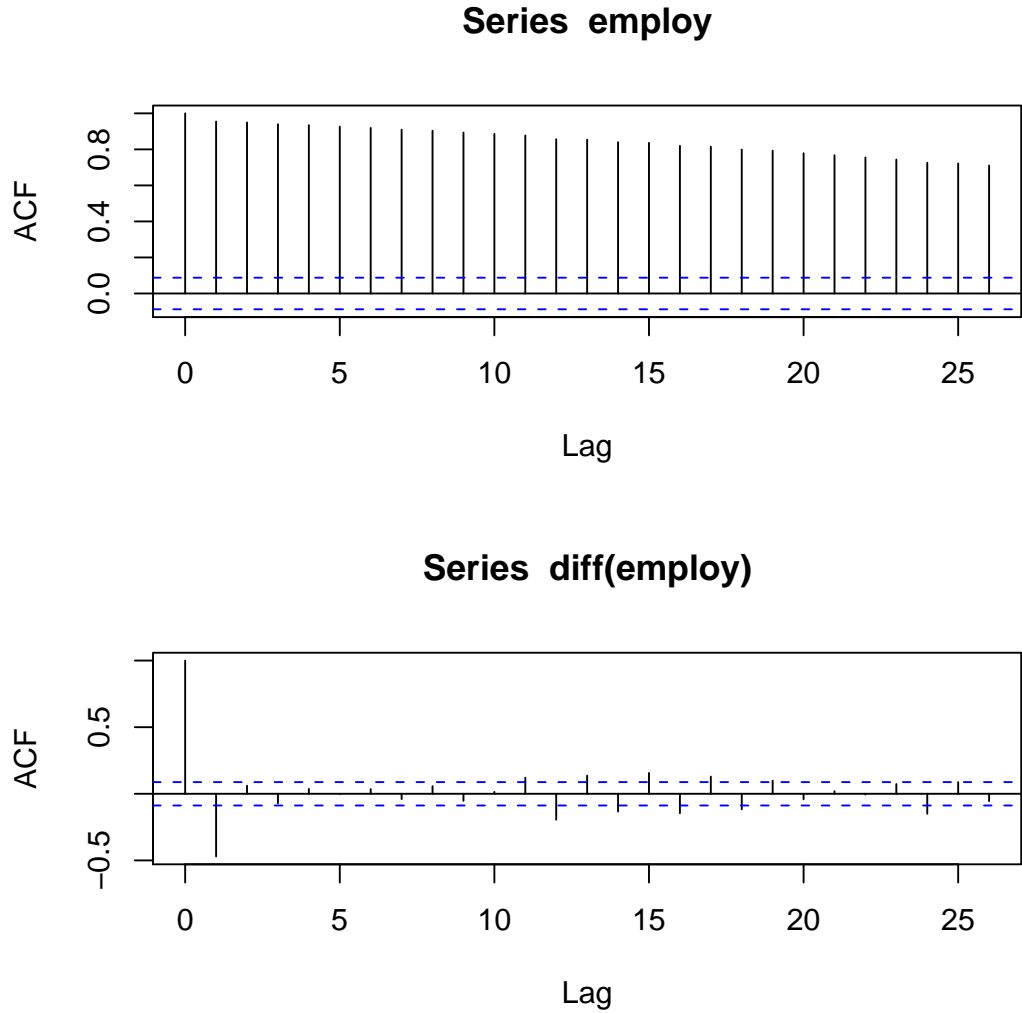


Figure 3: This plot shows the number of casualties in the UK from January 1969 to December 1984. The top plot shows the data along with a moving average smoothing estimate of the mean with 12 terms. The lower plot shows the data along with a moving average smoothing estimate of the mean with 3 terms.

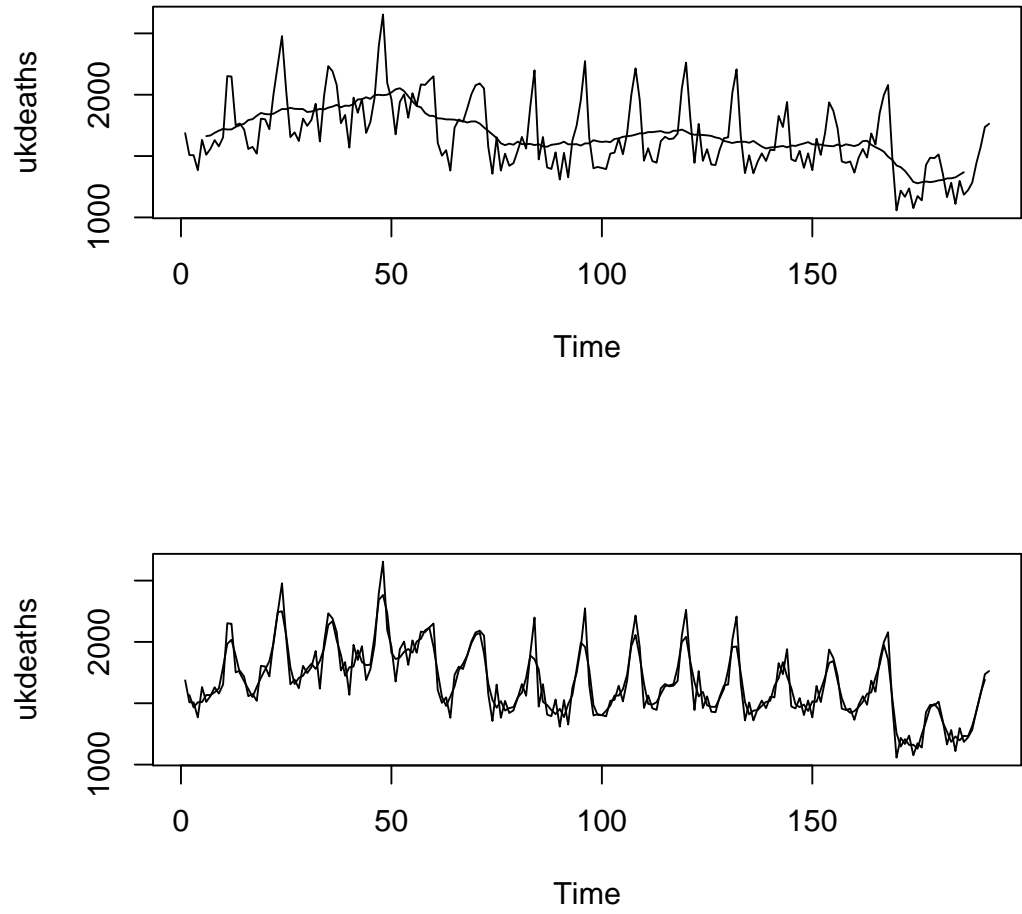


Figure 4: The top plot shows the residuals for the casualties data after a moving average smoothing estimate of the mean with 12 terms is removed. The lower plot shows the residuals for the casualties data after a moving average smoothing estimate of the mean with 3 terms is removed.

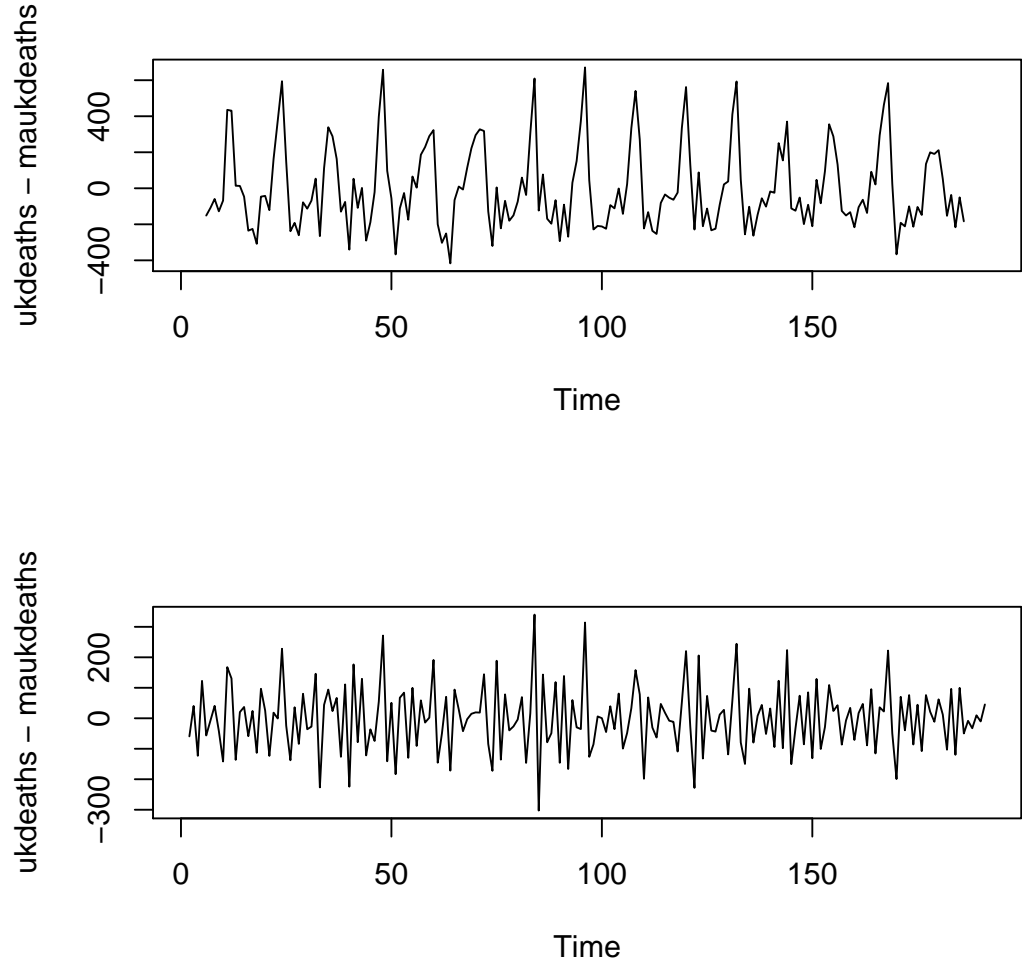


Figure 5: The top plot shows acf of the residuals for the casualties data after a moving average smoothing estimate of the mean with 12 terms is removed. The lower plot shows the acf of the residuals for the casualties data after a moving average smoothing estimate of the mean with 3 terms is removed.

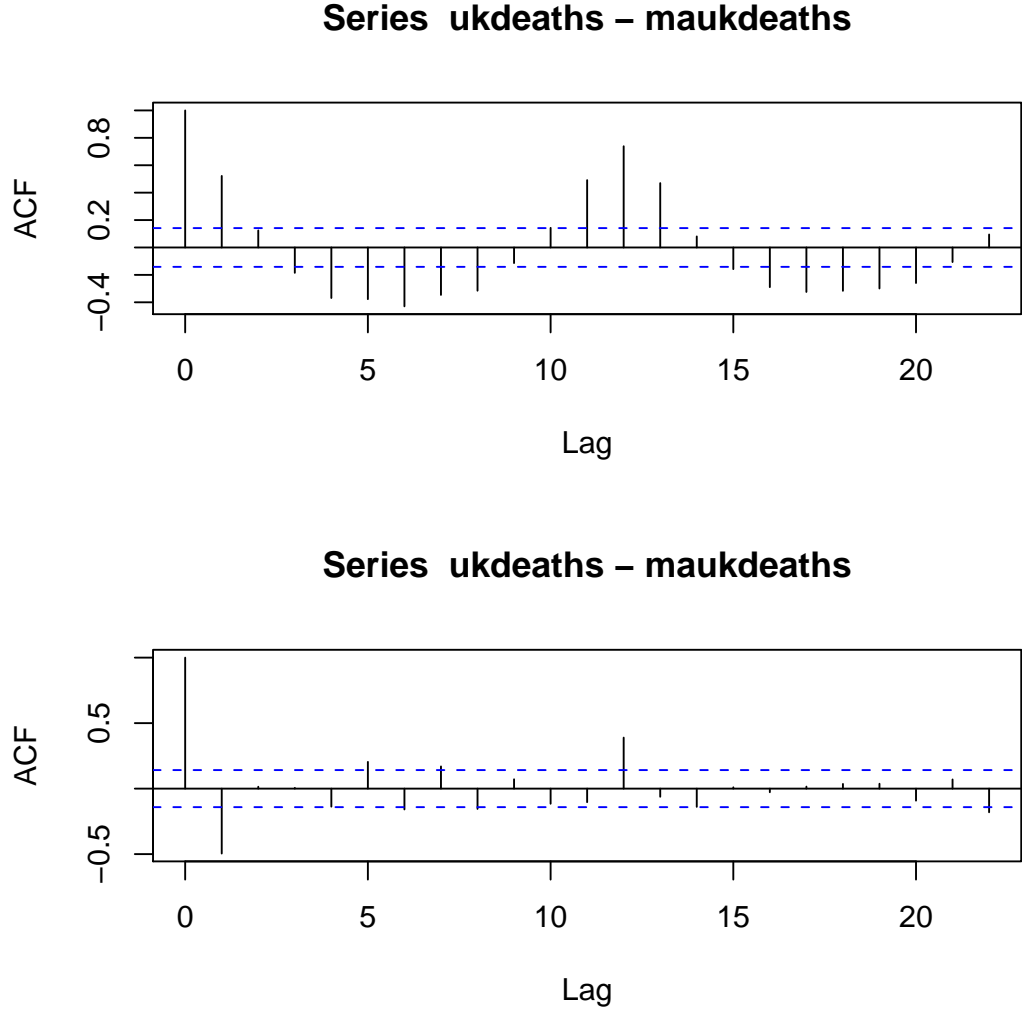


Figure 6: The plot shows periodogram of the residuals for the casualties data after a moving average smoothing estimate of the mean with 12 terms is removed.

