STAT 513
Homework 8

Due never

1. Calculate $P(|X - \mu| \geq k\sigma)$ for $X \sim Unif(0, 1)$ and $X \sim Exp(\lambda)$ and compare your answers to the bound from Chebychev's inequality.

2. If X is a random variable whose MGF exists, prove that $P(X \geq 0) \leq Ee^{tX}$ for all $t \geq 0$ for which the mgf is defined.

3. A random variable X is defined by $Z = logX$, where $EZ = 0$. Is EX greater than, less than, or equal to 1?

4. Let X be a binomial random variable with parameters n and p show that for $i > np$,

   (a) the minimum of $e^{-it}E\left[e^{tX}\right]$ occurs when t satisfies

   $$e^t = \frac{iq}{(n-i)p}$$

   (b) and

   $$P(X \geq i) \leq \frac{n^n}{i^i(n-i)^{n-i}} p^i (1-p)^{n-i}$$

5. If X is a Poisson random variable with mean λ show that for $i < \lambda$,

   $$P(X \leq i) \leq \frac{e^{-\lambda}(e\lambda)^i}{i^i}$$

6. Let X be a non-negative absolutely continuous random variable having a non-increasing density function Show that

   $$f(x) \leq \frac{2EX}{x^2}, \quad x > 0$$

7. If X has mean μ and standard deviation σ, the ratio $r = |\mu|/\sigma$ is called the measurement signal to noise ratio of X. Similarily, we define the relative deviation of X from its signal (μ) as

$$D = \left| \frac{X - \mu}{\mu} \right|$$

Prove that

$$P(D \leq \alpha) \geq 1 - \frac{1}{r^2 \alpha^2}$$