Exam 2 Review

1. The number of hours between successive train arrivals at the station is uniformly distributed on $(0, 1)$. Passengers arrive according to a Poisson process with rate λ per hour. Suppose a train has just left the station. Let X denote the number of people who get on the next train. Find $E[X^2]$.

2. After the bell has rung, late students straggle into class according to a Poisson process with rate $\lambda = 1/3$ minutes.

   (a) What is the variance for the arrival time of the first late student?

   (b) What is the mean time for the arrival of the first late student given that only one arrives in the first five minutes?

   (c) If the proportion of the class is 40% female and 60% male, what is the probability that the first late student was a woman followed by two men?

   (d) What is the probability that two female students arrive in the first five minutes given that three male students arrive in the same time period?

   (e) What is the variance of the arrival time of the third late male student?

3. A computer computer lab has three laser printers, two that are hooked to the network and one that is used as a spare. A working printer will function for an exponential amount of time with mean 20 days. Upon failure it is immediately sent to the repair facility and replaced by another machine if there is one in working order. At the repair facility machines are worked on by a single repairman who needs an exponential amount of time with mean 2 days to fix one printer. In the long run how often are there two working printers?

4. Consider a barbershop with two barbers and two waiting chairs. Customers arrive at a rate of 5 per hour. Customers arriving to a fully

occupied shop leave without being served. Find the stationary distribution for the number of customers in the shop, assuming that the service rate for each barber is 2 customers per hour.

5. Customers arrive at the Shortstop convenience store at a rate of 20 per hour. When two or fewer customers are present in the checkout line, a single clerk works and the average service time is 3 minutes. However, when there are three or more customers present, an assistant comes over to bag up the groceries and reduces the service time to 2 minutes. Assuming the service times are exponentially distributed, find the stationary distribution.

6. Customers arrive at a shipping office at times of a Poisson process with rate 3 per hour. The office was supposed to open at 8 AM but the clerk Oscar overslept and came in at 10 AM.

   (a) What is the probability that no customer came in the two hours before Oscar arrived?

   (b) If one customer did arrive between 8 and 10, what is the expected value of his wait for Oscar?

   (c) If no customer arrived before 10, what is the distribution of the time that Oscar must wait for the arrival of the first customer?

   (d) What is the variance of the time until the fifth customer arrives after 10 AM?

7. Suppose that the number of calls per hour to an answering service follows a Poisson process with rate 4.

   (a) What is the probability that fewer than two calls came in the first hour?

   (b) Suppose that six calls arrive in the first two hours, what is the probability exactly two arrived in the first hour and four in the second hour.

   (c) Suppose that the operator gets to take a break after she has answered ten calls. How long are her average work periods?

   Now, suppose that 3/4 of the calls are made by men and 1/4 by women.

   (d) Given that 3 men call in one hour, what is the probability that 2 women call in that hour?

(e) What is the probability that in one hour exactly 2 men and 3 women will call the answering service?

(f) What is the average time before 3 women call?

8. Edwin catches fish at time that correspond to a Poisson process with rate 3 per hour. Suppose 40% of the fish are salmon and 60% are trout. The trout weigh an average of 4 pounds with a standard deviation of 2 pounds. Find the mean and standard deviation of the total weight of trout he catches in two hours.