DEPARTMENT OF STATISTICS
Penn State University, Statistics 514 (Spring 2010)

Final Exam

Name (**Please Print**):

Instructions: This is a closed book, closed notes examination. You may refer to your summary notes during the exam. You must SHOW ALL YOUR WORK.

You may use these information in the following questions: If $Y \sim Gamma(\alpha, \beta)$, then $f(y|\alpha, \beta) = 1/\{\Gamma(\alpha)\beta^{\alpha}\}y^{\alpha-1}e^{-y/\beta}$, $y > 0$, where $E(Y) = \alpha\beta$ and $Var(Y) = \alpha\beta^2$. Also note that for a random sample $Y_1, \ldots, Y_n$ from $Gamma(1, \beta)$, $\sum_{i=1}^{n} Y_i \sim Gamma(n, \beta)$.

1. Let $X_1, \ldots, X_n$ be iid from $Gamma(1, 1/\theta)$, $\theta > 0$.

   a. (10 points) Find the mle estimator of θ.

   b. (10 points) Find the asymptotic distribution of the mle estimator of θ.

   c. (15 points) Find the UMVUE estimator of θ.
   Hint: Begin by considering the mle estimator.

   d. (5 points) Find the asymptotic distribution of the UMVUE estimator of θ.

2. (20 points) Let $X_1, \ldots, X_n$ be a random sample from $Gamma(1, \theta)$, $\theta > 0$. Derive the UMP size α test for testing $H_0 : \theta = 1$ versus $H_1 : \theta > 1$. After deriving at the critical region and arguing why the test is UMP, describe thoroughly how the constants in the critical region would be computed referring to specific distributions.

3. (25 points) Let $X_1, \ldots, X_n$ denote a random sample from a distribution that is $N(0, \theta)$, where the variance θ is an unknown positive number. Derive the likelihood ratio test for testing $H_0 : \theta = \theta'$ vs. $H_A : \theta \neq \theta'$. Derive at the critical region where you specify the explicit distribution of the test statistic under the null. Describe thoroughly how the constants in the critical region would be computed referring to specific distributions. Include full justification to receive full credit.

4. (15 points) Suppose $X_1, \ldots, X_n$ are iid from a $Gamma(3, 1/\theta)$ distribution with $\theta > 0$. If the prior density is $\theta \sim Gamma(1, 1/\lambda)$ with $\lambda > 0$, what is the Bayes estimate of θ corresponding to squared error loss?