STAT 513
Exam 1

Fall 2011

1. Three fair coins are tossed. Let Y_1 be the number of heads on the first two coins, and Y_2 be the number of tails on the last two coins.
   (a) (10 points) Find the joint distribution of Y_1 and Y_2.
   (b) (5 points) Find the conditional distribution of Y_2 given that $Y_1 = 1$.

2. (15 points) Let X and Y be independent standard normal random variables. Use the convolution method to show that $X+Y$ is normally distributed with mean 0 and variance equal to 2.

3. Let X have the uniform density, $f_X(x) = I_{\{0<x<1\}}$ and Y have the density $f_Y(y) = 3y^2 I_{\{0<y<1\}}$. We toss a fair coin. If heads comes up we observe X, and if tails comes up we observe Y.
   (a) (10 points) What is the probability that the outcome is less than 1/2?
   (b) (5 points) Given that the probability is less than 1/2, what is the probability that we observed X?

4. (15 points) Show that a random variable X with pdf

$$f_X(x) = \exp\left(x - e^x\right), \ -\infty < x < \infty$$

   has the moment generating function $M_X(t) = \Gamma(t+1)$ with domain $t > -1$.

5. (10 points) If A, B and C are mutually independent, then $A \cup B$ and C are independent. Prove this statement, or give a counter example if it is false.

6. (15 points) Divide a line segment of length L into two parts by selecting a point at random. Find the probability that the larger segment is at least four times the shorter. Assume a uniform distribution.

7. (15 points) If X is Beta one ($B_1(\mu, \nu)$), then

$$Y = \frac{X}{1 - X}$$

is Beta two ($B_2(\mu, \nu)$). Prove by deriving the pdf of Y.