

Statistics 416

Name: \_\_\_\_\_

Final Exam

December 17, 2007

Make sure to put all your answers in the space provided. Also, be sure to give complete answers and to show your work. Complete answers must include relevant calculations and sufficient justification for your solution.

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1. (15 pts) The number of hours between successive train arrivals at the station is uniformly distributed on  $(0, 1)$ . Passengers arrive according to a Poisson process with rate  $\lambda = 2$  per hour. Suppose a train has just left the station. Let  $X$  denote the number of people who get on the next train. Find  $Var(X)$ .

2. (15 pts) A manufacturing process may be described as follows: A part to be manufactured will begin the process by entering step 1. After step 1, 20% of the parts must be reworked, i.e. returned to step 1., 10% of the parts are thrown away, and 70% proceed to step 2. After step 2, 5% of the parts must be returned to step 1, 10% to step 2, 5% are scrapped, and 80% emerge to be sold for a profit.
- (a) Formulate a Markov chain model for this problem.
  - (b) If starting at state 1 at time zero, what is the probability of being in step 2 at time two?

3. (20 points) Two frogs are playing near a pond. When they are in the sun they get too hot and individually jump in the lake at rate 1 frog per hour. When they are in the lake they get too cold and each jump onto the land at rate 2. Let  $X(t)$  be the number of frogs in the sun.
- (a) Find the Q matrix for  $X(t)$ .
  - (b) If there are two frogs in the sun, what is the precise distribution of the time until one of the frogs jumps into the water? (i.e. Write the probability density for this time.)
  - (c) Find the stationary distribution for the chain.





5. (20 pts) Suppose that three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck.
- (a) Give the  $P$  matrix for the corresponding discrete time Markov chain. (Time here represents the  $n$ th vehicle passing.)
  - (b) Assume that a vehicle passes at the rate of one every two minutes according to a Poisson process. Give the rate matrix,  $Q$ , for the corresponding continuous time Markov chain.