Statistics 416   Name: _____

Final Exam

Make sure to put all your answers in the space provided. Also, be sure to give complete answers and to show your work. In other words, you need to not only answer the questions, but also to convince me of your answer.

1. (15 pts) Customers arrive at an automated teller machine at the times of a Poisson process with rate of 10 per hour. Suppose that the amount of money withdrawn on each transaction has a mean of 30 and a standard deviation of 20. What is the variance of the amount of money withdrawn in 8 hours?

2. (15 pts) Two people are working in a small office selling shares in a mutual fund. Each is either on the phone or not. Suppose salesman one is on the phone for an exponentially distributed amount of time with rate $\mu_1 = 2$. Salesman two is on the phone for an exponentially distributed mount of time with rate $\mu_2 = 3$. The rate for an individual salesman to stay of the phone is $\lambda = 3$.

   (a) Write down the Q matrix for a continuous time Markov chain representing WHICH salesman is on the phone. (The state space should be $\{0, 1, 2, 12\}$.)

   (b) Can one easily formulate a Markov chain, $X(t)$, which represents that number of salesman on the phone ?

(c) Find the stationary distribution for the chain.

3. (15 pts)In a True/False test paper the questions are arranged so that 3/4's of the time a True answer is followed by a True, while 2/3's of the time a False answer is followed by a False. Suppose there are 100 questions on the test. Approximately what fraction of the answers will be true? If you were told that the first question was True with probability 1/2, would the answer you've given be different? Explain.

4. (15 pts) Suppose that the following is the probability transition matrix for a Markov chain, X_i which has state space $0, 1, 2$.

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 1/3 & 0 & 2/3 \\ 0 & 1/3 & 2/3 \end{pmatrix}$$

(a) Is this Markov chain reducible? What does this imply about the existence of a stationary distribution?

(b) Suppose that the initial distribution, i.e. $P(X_0 = i)$, for the chain is $\alpha = (1/4, 1/4, 1/2)$. Find EX_2.

5. (10 pts) Customers arrive at the Shortstop convenience store at a rate of 20 per hour. When two or fewer customers are present in the checkout line, a single clerk works and the average service time is 3 minutes. However, when there are three or more customers present, an assistant comes over to bag up the groceries and reduces the service time to 2 minutes. Write down the Kolmogorov forward equations for this model.

Arrival rate: $\lambda = 20$ per hour.

Service rates: when $n \le 2$, service time 3 min $\Rightarrow \mu = 20$ per hour; when $n \ge 3$, service time 2 min $\Rightarrow \mu = 30$ per hour. Thus $\mu_1 = \mu_2 = 20$ and $\mu_n = 30$ for $n \ge 3$.

Kolmogorov forward equations:

$$P_0'(t) = -20\,P_0(t) + 20\,P_1(t)$$
$$P_1'(t) = -(20+20)\,P_1(t) + 20\,P_0(t) + 20\,P_2(t)$$
$$P_2'(t) = -(20+20)\,P_2(t) + 20\,P_1(t) + 30\,P_3(t)$$
$$P_n'(t) = -(20+30)\,P_n(t) + 20\,P_{n-1}(t) + 30\,P_{n+1}(t), \quad n \ge 3$$

6. (10 pts) Suppose there are two types of monomers, A and B. These can come together to form a complex C made up of two of the A monomers and one of the B monomers. If there are only two of the A monomers and one of the B this will occur at rate λ. If there is only one of the C complex, then it will break apart and become two of the A monomers and one of the B at rate μ.

   (a) If there are i of the A type, j of the B type, and k of the C type, what are the possible reactions along with their corresponding rates? (Assume i, j, and k are greater than or equal to 2.)

   (b) If there is one of the A type and one of the B type, what is the only possible reaction, and what is the distribution of time until the reaction occurs?

7. (20 pts) Suppose that a hospital is carrying out a clinical trial for a treatment technique for skin cancer. Patients with skin cancer arrive to the hospital at a rate of 5 per day. Sixty percent of these patients are female, and forty percent are male.

(a) Assume that you want 10 of each type of patient in order to have balance in the trial. What is the mean time and the variance of the time until 10 men are selected for the study? Is it possible for the hospital to select 10 men before 10 women for the trial?

(b) If 10 men are selected within 4 days, what is the probability that exactly 10 women are selected in the same two days?

(c) Suppose that only one man with skin cancer arrives in the fist day. Assuming the study started at midnight, was he more likely to arrive before noon or after noon? Explain your answer.

(d) If we continue to suppose only one man arrived in the first day, what is the expected number of patients with skin cancer to arrive in that first day?