

## Homework 10

1. Assume that a finite state continuous time Markov chain has a rate matrix that can be written in block form (as was done in class)

$$Q = \begin{pmatrix} A & B \\ 0 & 0 \end{pmatrix}$$

where  $A$  describes the transition rates between transient states and  $B$  describes the transition rates from transient to absorbing states. Also assume that the row vector  $a$  is of the length of the transient states and includes the initial probabilities for each of those states.

- (a) Find the general formula for the mean of the time to absorption.
- (b) Find the general formula for the variance of the time to absorption.

Now, assume for the set up of the previous problem that you have the following specific matrices (you may want to use R for this part). (The states are described by 0,1,2,3,4,5.)

$$A = \begin{pmatrix} -9 & 1 & 2 \\ 3 & -11 & 2 \\ 4 & 2 & -12 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

- (a) Give the mean value of the time to absorption if you are twice as likely to start in state one than in either state zero or state two.
- (b) Given that you start in state 1, what is the probability of being absorbed in state 4? What is the probability of being absorbed in state 5 if you have the initial values described in the previously described in (a).

2. Do problem 10.1 from Ross.
3. Do problem 10.2 from Ross.
4. Do problem 10.8 from Ross.