

Homework 5

1. Suppose that the offspring distribution is Poisson with mean $\lambda = 1.1$. Compute the extinction probabilities $u_n = P(X_n = 0)$ for $n = 0, 1, 2, 3, 4$.
2. One-fourth of the married couples in a far-off society have no children at all. The other three-fourths of couples have exactly three children, with each child equally likely to be a boy or a girl. What is the probability that the male line of descent of a particular husband will eventually die out? (For a particular male, what is the probability that there are no males in a particular generation?)
3. Similar to the last problem. One-fourth of the married couples in a far-off society have exactly three children. The other three-fourths of couples continue to have children until the first boy and then cease childbearing. Assume that each child is equally likely to be a boy or girl. What is the probability that the male line of descent of a particular husband will eventually die out?
4. Let $\phi_Z(s) = as^2 + bs + c$, where a, b, c are positive and $\phi(1) = 1$. Assume that the probability of extinction is u_∞ , where $0 < u_\infty < 1$. Prove that $u_\infty = c/a$.
5. Consider a branching process whose offspring follow the geometric distribution
$$P_k = (1 - p)p^k$$
for $k = 0, 1, 2, \dots$ for $0 < p < 1$. Determine the probability of eventual extinction.
6. 4.64 from Ross.
7. 4.66 from Ross.