Homework 9

1. Let $X(t)$ be a birth and death process with possible states $0, 1, ..., N$ and birth rate $\lambda_i = \alpha(N-i)$and $\mu_i = \beta i$. Find the stationary distribution for the chain.

2. Determine the stationary distribution for a birth and death process with rates $\lambda_i = \alpha(i+1)$ and $\mu_i = \beta i^2$.

3. Find the stationary distribution for a birth-death process with $\lambda_i = \theta < 1$ and $\mu_i = i/(i+1)$.

4. Consider the Markov chain whose state space is $0, 1, 2, 3$ and probability transition matrix is given by

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.4 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0.6 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Starting in state one, determine the probability that the Markov chain ends in state zero.

   (b) Determine the mean time to absorption when starting in either state 1 or 2.

5. Consider the Markov chain whose state space is $0, 1, 2, 3$ and probability transition matrix is given by

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.2 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.6 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Starting in state 2, determine the probability that the Markov chain ends in state 0.

(b) Determine the mean time to absorption when starting in state 2.

(c) Starting in state 2, determine the mean time that the process spends in state 1 prior to absorption and the mean time that the process spends in state 2 prior to absorption. Verify that the sum of these is the mean time to absorption.