STAT 513
Homework 2

Due 9.18.2011

Please read pages 13 through 19 of Casella and Berger starting on these questions.

1. Let $A_1, A_2, ..., A_n$ be arbitrary subsets of a set ω. Describe (explicitly) the smallest σ-field $\mathcal{F}$ containing $A_1, ..., A_n$. How many sets are there in $\mathcal{F}$? (Give an upper bound that is attainable under certain conditions.) List all the sets in $\mathcal{F}$ when $n = 2$.

2. Let Ω be the natural numbers. Give an example of a σ-field for this sample space. Give the smallest σ-field that contains the even natural numbers.

3. A man is given n keys of which only one fits his door. He tries them successively. This procedure make require $1, 2, ..., n$ trials. Show that each of these n outcomes has probability n^{-1}.

4. Player A throws six dice and wins if he scores at least one ace. Player B throws twelve dice and wins if he scores at least two ace. Who has the greater probability of winning?

5. A car is parked among N cares in a row, not at either end. On his return, the owner finds that exactly r of the N places are still occupied. What is the probability that both neighboring places are empty.

6. A red card is removed from a deck of 52 cards; 13 cards are then drawn and found to be the same color. Show that the conditional probability that all will be black is equal to $\frac{2}{3}$.

7. Each of 3 boxes, identical in appearance, has 2 drawers. Box A contains a gold coin in each drawer; box B contains a silver coin in each

drawer; box C contains a gold coin in one drawer and a silver coin in the other. A box is chosen, one of its drawers is opened, and a gold coin is found.

(a) What is the probability that the other drawer contains a silver coin? Write out the probability space of the experiment. Why is it fallacious to reason that the probability is $\frac{1}{2}$ that there will be a silver coin in the second drawer, since there are 2 possible types of coins, gold or silver, that may be found there?

(b) What is the probability that the box chosen was box A? Box B? Box C?

8. Consider an urn that contains 5 white and 7 black balls. A ball is drawn and its color is noted. It is then replaced; in addition, 3 balls of the color drawn are added to the urn. A ball is then drawn from the urn. Find the probability that (i) the second ball drawn will be black, (ii) both balls drawn will be black.

9. A closet contains n pairs of shoes. If $2r$ shoes are chosen at random (with $2r < n$), what is the probability that there will be (a) no complete pair, (b) exactly one complete pair, (c) exactly two complete pairs among them?

10. (a) If n balls are placed at random into n cells, find the probability that exactly one cell remains empty.

(b) In a certain (rather traditional) family, four girls take turns at washing dishes. Out of a total of four breakages, three were caused by the youngest girl, and she was thereafter called clumsy. Was she justified in attributing the frequency of her breakages to chance? Discuss the connection with random placements of balls.