Math/Stat 414 Exam 1 Name: ____________________

Instructions: Please read!

1. Do all work on this exam packet. It is okay to leave your answer unsimplified, as in $0.56\frac{15!}{6!9!}$ or $14e - 20$. This means that no calculator is needed.
2. Show all work for full credit! Small mistakes in arithmetic will not reduce credit if you show work; conversely, even a correct answer could get no credit without supporting work.
3. I will award partial credit where appropriate.

---

1. The monthly worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that there will be at least 2 such accidents in the next month? Explain any assumptions that you may need to make.

   solution: There are many (at least many thousands) of plane flights per month (large n). Clearly, the probability of any particular flight crashing is very small (i.e. small p). If we assume that each flight has the same probability of crashing and the events that each flight crashes are independent, then the number of crashes is binomial. (Both of these assumptions are dubious. Flights with the same crews and planes have a more similar p than others. Also, the same crew flying two flights would make those flights dependent. However, given the large number of flights that have little to do with one another–the assumptions may not be too unrealistic.) Since the number of crashes is binomial with very large n and very small p. We model the number of monthly crashes using a Poisson distribution with $\lambda = 3.5$. So, the probability there are at least 2 such crashes is

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - e^{-3.5} - 3.5e^{-3.5}$$

2. Seven balls are randomly withdrawn from an urn that contains 12 red, 16 blue, and 18 green balls. Find the probability that

   (a) 3 red, 2 blue, and 2 green balls are withdrawn

   solution: Count the ways to pull these number of balls and divide by the number of ways to pull 7 balls from a total of 46 in the urn.

$$\frac{\binom{12}{3}\binom{16}{2}\binom{18}{2}}{\binom{46}{7}}$$

   (b) the first three are red, the next two are blue and the last two are green

   solution: Think of this as three separate experiments done in sequence with the previous experiments altering your options.

$$\frac{\binom{12}{3}}{\binom{46}{3}}\frac{\binom{16}{2}}{\binom{43}{2}}\frac{\binom{18}{2}}{\binom{41}{2}}$$

   (c) at least 2 red balls are withdrawn

   solution: It will be easier to calculate $1 - P(1\ red) - P(0\ red)$. Also, note there are

$$P(\geq\ 2\ red) = 1 - P(1\ red) - P(0\ red) = 1 - \frac{\binom{12}{1}\binom{34}{6}}{\binom{46}{7}} - \frac{\binom{12}{0}\binom{34}{7}}{\binom{46}{7}}$$

   (d) all withdrawn balls are the same color

   solution: Since each color would be disjoint, $P(all\ balls\ same\ color) = P(all\ balls\ blue) + P(all\ balls\ red) + P(all\ balls\ green)$. So, the answer is

$$P(all\ balls\ same\ color) = \frac{\binom{16}{7}\binom{30}{0}}{\binom{46}{7}} + \frac{\binom{12}{7}\binom{34}{0}}{\binom{46}{7}} + \frac{\binom{18}{7}\binom{28}{0}}{\binom{46}{7}}$$

3. Suppose $Y = 2X + 3$. Also, suppose that $EY = 9$ and $EY^2 = 100$.

   (a) Find EX.

   solution: Note that $EY = 2EX + 3$, a fact from class/book. So, $9 = 2EX + 3$, which leads to $EX = 3$.

   (b) Find $Var(X)$.

   solution: Note that $Var(Y) = Var(2X + 3) = 4Var(X)$, which leads to $Var(Y) = 4Var(X)$. Since $Var(Y) = EY^2 - (EY)^2$, we see that $Var(Y) = 100 - 81 = 19$. So, $Var(X) = 19/4$.

4. Suppose that 5 percent of men and 0.25 percent of women in a population are color blind.

   (a) Assuming a population is 50 percent men and 50 percent women, what is the proportion of the population that is color blind?
   solution:Law of total probability gives

$$P(cb) = P(cb|female)P(female) + P(cb|male)P(male)$$

   Filling in the values.

$$P(cb) = 0.0025(0.5) + 0.05(0.5)$$

   (b) Continuing to assume an even split in the total population between men and women, what is the probability that a randomly selected person from among the color blind population is male?
   solution: Use Bayes' rule

$$P(male|cb) = \frac{P(cb|male)P(male)}{P(cb|female)P(female) + P(cb|male)P(male)}$$

   Filling in the values,

$$P(male|cb) = \frac{0.05(0.5)}{0.0025(0.5) + 0.05(0.5)}$$

5. When coin 1 is flipped, it lands on heads with probability 0.4; when coin 2 is flipped, it lands on heads with probability 0.7. One of these coins is randomly chosen and flipped 10 times.

   (a) What is the probability that the coin lands on heads on exactly 7 of the 10 flips?

   solution:

   $$P(7\ heads) = P(7\ heads|coin\ 1)P(coin\ 1)+P(7\ heads|coin\ 2)P(coin\ 2)$$

   Filling in the values and noting that the conditional distribution of the number of heads given coin 1 or coin 2 is binomial,

   $$P(7\ heads) = \binom{10}{7}0.4^7 0.6^3 0.5 + \binom{10}{7}0.7^7 0.3^3 0.5$$

   (b) Given that the first two of the ten flips are both heads, what is the conditional probability that exactly 7 of the 10 flips land on heads? First, if we know the probability of a head is p, then

   $$P(7\ heads|first\ two\ heads) = \frac{P(Y=5\ and\ first\ two\ heads)}{P(first\ two\ heads)} = \frac{P(Y=5)P(first\ two\ heads)}{P(first\ two\ heads)} = P(Y=5)$$

   where Y is a binomial random variable with $n = 8$ and is independent of the first two flips. So,

   $$P(7\ heads|first\ two\ heads) = P(Y=5) = P(Y=5|coin\ 1)P(coin\ 1)+P(Y=5|coin\ 2)P(coin\ 2)$$

   Filling in the values

   $$P(7\ heads) = \binom{8}{5}0.4^5 0.6^3 0.5 + \binom{8}{5}0.7^5 0.3^3 0.5$$