

Quiz 1

Make sure to put all your answers in the space provided. You are allowed to have only a writing utensil. **No** calculators, cell phones, scrap paper, etc. Also, be sure to give complete answers and to show your work. In other words, you need to not only answer the questions, **but also to convince me of your answer.**

1. (5 pts) A dance class consists of 22 students, of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?

solution: There are $\binom{12}{5}$ ways to select five men (without order) and $\binom{10}{5}$ ways to select five women (without order). Now, we need to create couples with the men and women selected. Order the men. How many ways can we then assign the women? $5!$. So, there are

$$\binom{12}{5} \binom{10}{5} 5!$$

ways to create the couples.

2. (5 pts)An investor has 20 thousand dollars to invest among 4 possible investments. Each investment must be in units of a thousand dollars. If the entire 20 thousand is to be invested, how many different investment strategies are possible? How many are possible if at least some money must be invested in each of the investments?

solution: This is equivalent to answering the question: how many non-negative integer solutions are there for the equation (in thousands)

$$x_1 + x_2 + x_3 + x_4 = 20$$

So, for the first question some investments could be left with zero. So, there are

$$\binom{n+r-1}{r-1} = \binom{23}{3}$$

solutions to the equations and thus that many investment strategies.

If some money must go to each investment, then each variable must be greater than or equal to one. So, there are

$$\binom{n-1}{r-1} = \binom{19}{3}$$

solutions to the equations in this case and thus that many investment strategies.