Statistics 414 Name: _____

Quiz 4

Make sure to put all your answers in the space provided. You are allowed to have only a writing utensil. **No** calculators, cell phones, scrap paper, etc. Also, be sure to give complete answers and to show your work. In other words, you need to not only answer the questions, **but also to convince me of your answer.**

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1. (5 pts) Three dice are rolled. Assuming that each outcome of the three dice is equally likely, find the probability mass function for X, the sum of the values of the dice.

   solution:We need to find $P(X = x)$ for a fixed x. Note the symmetry of the problem. So, $P(X = 3) = P(X = 18)$, $P(X = 4) = P(X = 17)$, $P(X = 5) = P(X = 16)$, $P(X = 6) = P(X = 15)$, $P(X = 7) = P(X = 14)$, and $P(X = 8) = P(X = 13)$. For each of these Each outcome of the roll has probability $1/6^3$, so we need to count the number of outcomes that sum to x on 3 dice. This is equivalent to asking how many strictly positive solutions are their to the equation $y_1 + y_2 + y_3 = x$. From chapter 1, we recall there are $\binom{x-1}{3-1}$ ways to do this. So,

   $$P(X = x) = \binom{x-1}{2}\frac{1}{6^3}$$

   for $x = 3, 4, 5, .., 8$. So, we need only to cover the case of $x = 9$ and $x = 10$. For the case of $x = 9$, we need to only subtract from the number of solutions to the equations described above–the ones that include a 7–since the maximum face of a die is six. There are three of these.

   $$P(X = 9) = (\binom{x-1}{2} - 3)\frac{1}{6^3}$$

   This also the probability of $X = 12$. For $P(X = 10)$, we may make a similar argument. Take all the solutions minus those that involve an 8 (there are three of these) or those that involve a 7 (there are six of these).

   $$P(X = 10) = (\binom{x-1}{2} - 9)\frac{1}{6^3}$$

   This is also the probability that $X = 11$.

2. (5 pts) If X has the cumulative distribution function $F(x)$, what is the distribution function of e^X expressed in terms of F? Remember to justify your answer.

   solution: $F(x) = P(X \leq x)$. So, $G(y) = P(e^X \leq y) = P(X \leq \log y) = F(\log y)$ (for $y > 0$).