Statistics 414 Name: ____________________

Quiz 5

Make sure to put all your answers in the space provided. You are allowed to have only a writing utensil. **No** calculators, cell phones, scrap paper, etc. Also, be sure to give complete answers and to show your work. In other words, you need to not only answer the questions, **but also to convince me of your answer.**

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1. (5 pts) You have \$1,000, and a certain commodity presently sells for \$2 an ounce. suppose that after one week the commodity will sell for either \$1 or \$4, with these possibilities equally likely. If your objective is to maximize the expected amount of money that you possess at the end of the week, what strategy should you employ?

   solution: Let a be the amount of money from the \$1,000 used to buy the commodity presently. Also, let X be the price of the commodity after a week. So, X is a RV with value \$1 with probability 1/2 and value \$4 with probability 1/2. To maximize the amount of money, you should sell whatever commodity is bought at the present time. So, we should maximize

   $$1000 - a + X\frac{a}{2}$$

   In words, this is the amount of money not spent on the commodity plus the price after a week times the amount of commodity purchased. So, the expected value of this is

   $$E(1000 - a + X\frac{a}{2}) = 1000 - a + \frac{a}{2}EX$$

   where $EX = 5/2$. So, the expected amount of money is $1000 - a + 5a = 1000 + 4a$. To maximize the amount of money, the investor should use the entire $1,000$ to buy the commodity from the beginning. (Note that the investor could certainly lose money, so expected gain may not be the only thing to consider.)

2. (5 pts) The expected number of typographical errors on a page of a certain magazine is 0.2. What is the probability of 3 or more typographical errors? Explain your reasoning.

solution: The number of letters on a page is high, certainly on the order of a thousand. If the expected number of errors is 0.2, then typographical errors are also rare. With a large number of Bernoulli trials at each letter and p quite small, a Poisson random variable (X) is an appropriate way to model the number of errors with $\lambda = 0.2$.

$$P(X \geq 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2) = 1 - e^{-0.2} - 0.2e^{-0.2} - \frac{(0.2)^2}{2}e^{-0.2}$$