

## Quiz 6

Make sure to put all your answers in the space provided. You are allowed to have only a writing utensil. **No** calculators, cell phones, scrap paper, etc. Also, be sure to give complete answers and to show your work. In other words, you need to not only answer the questions, **but also to convince me of your answer.**

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1. ( 5 pts ) The density function of  $X$  is given by

$$f(x) = (a + bx^2)I_{\{0 < x < 1\}}$$

If  $E[X] = \frac{3}{5}$ , find  $a$  and  $b$ .

*solution:*  $a$  and  $b$  must satisfy

$$1 = \int_0^1 a + bx^2 dx = a + \frac{1}{3}b$$

and

$$\frac{3}{5} = \int_0^1 ax + bx^3 dx = \frac{1}{2}a + \frac{1}{4}b$$

Substituting the first into the second we have

$$\frac{3}{5} = \frac{1}{2} \left( 1 - \frac{1}{3}b \right) + \frac{1}{4}$$

leading to  $b = 6/5$  substituting this into the first equation gives  $a = 3/5$ .

2. ( 5 pts ) A point is chosen at random on a line segment of length  $L$ . Find the probability that the ratio of the shorter to the longer segment is less than  $\frac{1}{4}$ .

*solution:* We need to find

$$P\left(\frac{X}{L-X} < 1/4 \text{ or } \frac{L-X}{X} < 1/4\right)$$

Given the symmetry of the problem this is equivalent to finding

$$2P\left(\frac{X}{L-X} < 1/4\right)$$

So, let's find

$$P\left(\frac{X}{L-X} < \frac{1}{4}\right) = P\left(X < \frac{1}{4}(L-X)\right) = P\left(X < \frac{1}{5}L\right)$$

This is equal to the integral

$$\int_0^{1/5L} \frac{1}{L} dx = \frac{1}{5}$$

So, the final answer is  $\frac{2}{5}$