

Quiz 7

Make sure to put all your answers in the space provided. You are allowed to have only a writing utensil. **No** calculators, cell phones, scrap paper, etc. Also, be sure to give complete answers and to show your work. In other words, you need to not only answer the questions, **but also to convince me of your answer.**

1. (5 pts) The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a previously owned radio that has been in use for four years, what is the probability that it will be working after an additional eight years.

solution: Let X be the time for the radio to fail. So, X has the distribution function

$$F_X(x) = 1 - e^{-x/8}$$

Thus,

$$P(X > x) = e^{-x/8}$$

Now, the probability in which we are interested is the probability that the radio lasts eight additional years past the four years it has already been in use. In symbols,

$$P(X > 12 | X > 4) = \frac{P(X > 12, X > 4)}{P(X > 4)} = \frac{P(X > 12)}{P(X > 4)} = \frac{e^{-12/8}}{e^{-4/8}} = e^{-1}$$

This is essentially the memoryless property of the exponential.

2. (5 pts) If X has uniform distribution from zero to one, find the density of $Y = -\log X$.

solution: Note that

$$f_X(x) = I_{\{0 < x < 1\}}$$

and also that Y can take on any value in the non-negative reals. Now,

$$F_Y(y) = P(Y \leq y) = P(-\log X \leq y) = P(X > e^{-y})$$

So, using the density for X we have

$$F_Y(y) = \int_{e^{-y}}^{\infty} I_{\{0 < x < 1\}} dx = \int_{e^{-y}}^1 dx = 1 - e^{-y}$$

Taking the derivative with respect to Y , we get

$$f_Y(y) = \frac{d}{dy} F_Y(y) = e^{-y}$$

So, Y has a standard exponential random distribution.