Statistics 414 Name: ____________________

Quiz 9/10

Make sure to put all your answers in the space provided. You are allowed to have only a writing utensil. **No** calculators, cell phones, scrap paper, etc. Also, be sure to give complete answers and to show your work. In other words, you need to not only answer the questions, **but also to convince me of your answer.** The first two questions are Quiz 9. The second two are Quiz 10. The grades for these will be recorded separately.

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1. (5 pts) Find the moment generating function for X, a binomial random variable with parameters n and $0 < p < 1$.

   solution:

$$M(t) = Ee^{tX} = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x (1-p)^{n-x} = \sum_{x=0}^{n} \binom{n}{x} (pe^t)^x (1-p)^{n-x}$$

   By the binomial theorem,

$$\sum_{x=0}^{n} \binom{n}{x} (pe^t)^x (1-p)^{n-x} = (1 - p + pe^t)^n$$

2. (5 pts) Find the mean and variance of X, which has moment generating function

$$M_X(t) = e^{t^2}$$

solution: Find the first and second moments by taking the derivative of the mgf twice.

$$M'_X(t) = 2te^{t^2}$$

and

$$M''_X(t) = 4t^2e^{t^2} + 2e^{t^2}$$

So,

$$EX = M'_X(0) = 0$$

and

$$EX^2 = M''_X(0) = 2$$

Since $EX = 0$, the variance is equal to the second moment, so

$$VarX = 2$$

3. (5 pts) In a class, there are 4 freshman boys, 6 freshman girls, and 6 sophomore boys. How many sophomore girls must be present if gender and class are to be independent when a student is selected at random?

solution: Let x be the number of sophomore girls present. Note that

$$P(freshmen\ \ boys) = \frac{4}{16+x}$$

and

$$P(freshmen)P(boys) = \frac{10}{16+x}\frac{10}{16+x}$$

In order for there to be independence, these two things must be equal. Find x such that

$$\frac{4}{16+x} = \frac{100}{(16+x)^2}$$

or

$$4(16+x) = 10$$

So, $x = 9$.

4. (5 pts) Two coins are flipped. The first coin will land on heads with probability 0.6, the second with probability 0.7. Assume that the results of the flip are independent, and let X equal the total number of heads that result. Find

   (a) $P(X = 1)$

   (b) EX

   solution:

   (a)
   $$P(X = 1) = 0.6(0.3) + 0.4(0.7) = 0.18 + 0.28 = 0.46$$

   (b)
   $$EX = 0P(X = 0) + 1P(X = 1) + 2P(X = 2)$$

   So, we need to find $P(X = 2) = 0.6(0.7) = 0.42$.

   $$EX = 0.46 + 2(0.42) = 1.3$$