

**Instructions: Please read!**

1. Do all work on this exam packet. It is okay to leave your answer unsimplified, as in  $0.56 \frac{15!}{6!9!}$  or  $14e - 20$ . This means that no calculator is needed.
  2. Show all work for full credit! Small mistakes in arithmetic wont reduce credit if you show work; conversely, even a correct answer could get no credit without supporting work.
  3. I will award partial credit where appropriate.
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1. Random variable  $X$  which has density

$$f(x) = \frac{1}{2}x^3, \quad -1 < x < 1$$

- (a) Find the variance of  $X$

- (b) Find the density of  $Y = X^2$ . Be sure to note the support of the resulting density.

2. Let  $T$  be uniform in the interval  $(0, 2)$  and  $S$  be independent of  $T$  and also uniform in the interval  $(0, 2)$ .

(a) Find  $P(Y(X + 1) < 1)$

(b) Find the moment generating function for  $T$ . **SHOW YOUR WORK.** This means you need to start with the definition of the moment generating function and showing the steps to arrive at the final answer.

- (c) Find the moment generating function for  $T + S$ . Does this correspond to the moment generating function for a distribution we know?

3. Let  $X$  have density

$$f_X(x) = \frac{3}{8}x^2, \quad 0 < x < 2$$

and  $Y$  be independent of  $X$  with density

$$f_Y(y) = \frac{3}{8}y^2, \quad 0 < y < 2$$

Find the density of the sum  $X + Y$  using the convolution method.

4. Let  $X$  and  $Y$  have joint density

$$f(x, y) = \frac{12}{5}y(2 - y - x), \quad 0 < x < 1, \quad 0 < y < 1$$

(a) Find  $P\left(X > \frac{1}{2} \mid Y = \frac{1}{2}\right)$ .

(b) Find  $E[XY]$ .