Solutions to Homework 1

MATH/STAT 414: Spring 2015

1. We will use the basic principle of counting to solve this problem.

   a The first two spaces reserved for letters can be filled in 26 ways, each. The last 5 spaces reserved for numbers can be filled in 10 ways each. Therefore, the possible number of 7-place license plates that can be made are:

   $$(26 * 26) * (10 * 10 * 10 * 10 * 10) = 67,600,000$$

   b In this case, we have 26 options to fill the first place. However, for the second place, only 25 letters are available to choose from. Similarly, the first of the 5 places reserved for numbers can be filled by 1 of the 10 numbers, the second by 1 of the 9 remaining numbers and so on. Hence, the possible number of 7-place license plates that can be made are:

   $$(26 * 25) * (10 * 9 * 8 * 7 * 6) = 22,464,000$$

2. The first worker can be assigned to any one of the 20 jobs. The next worker, however, can only be assigned to one of the remainin 19 jobs. The third worker can be assigned to one of the 18 jobs, and so on. So the number of different assignments possible here are: **20**!

3. We can solve this problem using the basic principle of counting.

   For all possible area codes, the first digit can be chosen in 8 ways, the second in 2 and the last one in 9 ways. Therefore, the number of ways to form a area code are:

   $$8 * 2 * 9 = 108$$

   In the second case, since the first digit is already fixed, we can only choose the remaining 2 digits as specified above. We have, $2 * 9 = 18$ ways of choosing the area code.

4. **a** By the same logic used in answering Question 2, we can think the following way. A given seat in the row can be filled by any one of the 6 persons. For each of those, the next seat can be filled by one of the 5 remaining students and so on. So 3 boys and 3 girls; each uniquely identifiable, can sit in a row in $\mathbf{6}! = 720$ ways

   b If the boys and girls must sit together, we can consider their groups as one entity each. So in a row, there are only 2! ways in which they can sit together. Additionally, within the 3 seats occupied by girls or boys, they can be seated in 3! ways each. Therefore the total number of seating arrangements are:

$$2! * 3! * 3! = 72$$

   c If the boys must sit together and the girls can occupy the remaining seats in any way possible, we can consider this to be an arrangement of 4 entities. There are 4! ways of arranging those. Additionally, the 3 boys can arrange themselves in 3! ways, in case of each arrangement. Hence the total number of ways in which everyone can be seated is:

$$4! * 3! = 144$$

   d If no two people of the same sex are allowed to sit beside each other, only two arrangements are possible and they are as denoted below: **GBGBGB** or **BGBGBG**. Within each of these arrangements, the boys and girls can seat themselves in 3! ways each. Therefore the total possible seating arrangements are:

$$2 * 3! * 2! = 72$$

5. Since there is no restriction on the arrangement, they colors of the blocks are irrelevant. The blocks can be arranged in any of the **12**! ways.

6. We can work through this problem in a simple way. If we choose one person to begin with, that person will shake 19 hands. The next person can shake only 18, since the handshake of first and second person is already accounted for. In this example, we will add these handshakes instead of multiplying them because they do not depend on each other. Each person shakes hands with every other person, there is no question of permutations or combinations. Hence the total number of handshakes is:

$$1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16+17+18+19 = 190$$

7. In this question, we are not conerned about the ordering or positioning. Therefore we will use the idea of combinations.

a If both books of the same subject are to be sold, they can both be either of the 6 math books, 7 science books or 4 economics books. Therefore the total number of choices can be written as:

$$\binom{6}{2} + \binom{7}{2} + \binom{4}{2}$$

b If both books have to be on a different a different subject, he can either sell a combination of math-science, science-economics or math-economics book. The number of ways to choose a math-science combination is equal to, choosing 1 out of 6 math books and choosing 1 of the science books for each choise of a math book. Hence the total number of ways of choosing 2 books to sell can be given as:

$$(\binom{6}{1} * \binom{7}{1}) + (\binom{7}{1} * \binom{4}{1}) + (\binom{6}{1} * \binom{4}{1})$$

8. Let us first note that there are $\binom{8}{5}$ ways of choosing the 5 friends who will be invited to the party. Now we will consider the two cases given in the question.

a Similar to the problem in the textbook, we can note that there are $\binom{2}{2} * \binom{6}{3}$ combinations of friends which will include both the feuding friends. This is true because $\binom{2}{2}$ is the number of ways of choosing both (2) of the 2 feuding friends and choosing 3 out of the remaining 6 friends. Hence the total number of ways of choosing 5 friends is:

$$\binom{8}{5} - \binom{2}{2} * \binom{6}{3} = 36$$

b If two of the friends will only attend together, they can either be invited or not. If they are to be invited, both of them can be chosen in $\binom{2}{2}$ ways and the remaining 3 friends will be chosen from out of 6 friends. Alternatively, $\binom{2}{0}$ is the number of ways in which neither of them will be chosen and the 5 friends to be invited will be chosen from among 6 friends. Therefore the total number of choices are:

$$(\binom{2}{2} * \binom{6}{3}) + (\binom{2}{0} * \binom{6}{5}) = 26$$

9. This question requires us to calculate the multinomial coefficient with $n = 12, n_1 = 3, n_2 = 4$ and $n_3 = 5$. This can be written as:

$$\frac{n!}{n_1!n_2!n_3!} = \frac{12!}{3!4!5!} = 27720$$