Homework 2: solutions

STAT 414: Spring 2015

1) - EF: 1 dice lands on 1 and other on an even number

- $E \cup F$: Sum of dice is odd OR at least 1 dice lands on 1 OR both

- FG: $(1,4), (4,1)$

- EF^c: Sum of dice is odd but none of the dice shows 1

- EFG: Since sum of 5 is contained in sum of dice being an odd number, $EFG = FG$: $(1,4), (4,1)$

2)

a) This sample space represents every possible number of tosses $(1, 2, 3, 4 \ldots\ldots)$ such that every toss except the last one is a tail (represented by 0)

b) <u>A wins</u>: Subset of sample space such that $(1 + 3k)^{th}$ toss is a head $(k = 0, 1, 2 \ldots)$ and all previous tosses are tail (or zero)

<u>B wins</u>: Similar to above except $(2 + 3k)^{th}$ toss is a head

<u>$(A \cup B)^c$</u> $= A^c \cap B^c$ = A does not win AND B does not win
$\equiv$ C wins : $(3k)^{th}$ toss is a head

3) $P(A) = 0.3$, $P(B) = 0.5$, $P(A \cap B) = 0$, Since they are mutually exclusive

- $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8$
- $P(A \cap B^c) = P(A) = 0.3$ Since mutually exclusive, if B does not happen, it does not affect A
- $P(A \cap B) = 0$

4) Let event A : smoking cigarettes and event B : smoking cigars

$\therefore P(A) = 0.28, \quad P(B) = 0.07, \quad P(A \cap B) = 0.05$

a) $P(A^c \cap B^c) = P(A \cup B)^c$ By D'Morgan's law

$= 1 - P(A \cup B)$

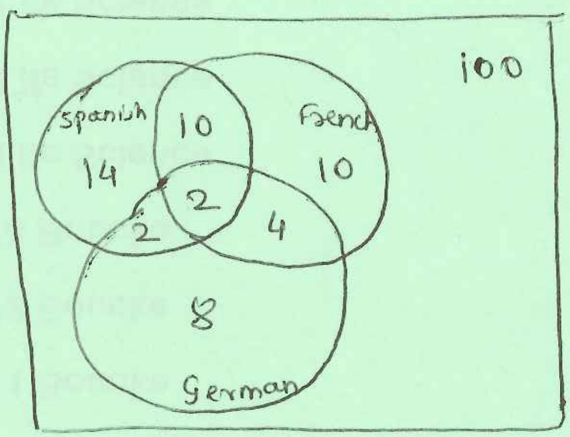
$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 0.7$

b) $P(A^c \cap B) = P(B) - P(A \cap B)$ By proposition 4.3

$= 0.02$

5)



Based on venn diagram,

a) $P(A \cup B \cup C)^c = 0.5$

b) $P(\text{Exactly one language}) = 0.32$

c) P(at least 1 of the 2 chosen students is taking a language) = 1 − P(None of the 2 chosen students is taking a language class)

$= 1 - \frac{{}^{50}C_2}{{}^{100}C_2}$

67) a) $P(\text{No 2 are alike}) = \dfrac{6 \times 5 \times 4 \times 3 \times 2}{6^5}$

b). $P(\text{one pair}) = \dfrac{6 \times {}^5C_2 \times 5 \times 4 \times 3}{6^5}$ {6 possible choices and 2 positions out of 5 to locate pair}

c) $P(\text{2 pairs}) = \dfrac{6 \times {}^5C_2 \times 5 \times {}^3C_2 \times 4}{{}^2C_1 \; 6^5}$ {6 choices for first paired # & $5C_2$ positions for that. 5 choices & $3C_2$ positions for 2nd paired #. Divide by $^2C_1$ to take care of doubling)

d) $P(\text{3 alike}) = \dfrac{6 \times {}^5C_3 \times 5 \times 4}{6^5}$

e) $P(\text{full house}) = P(\text{3 alike and 2 in pair})$

$= \dfrac{6 \times {}^5C_3 \times 5 \times {}^2C_2}{6^5}$

f) $P(\text{4 alike}) = \dfrac{6 \times {}^5C_4 \times 5}{6^5}$

g) $P(\text{5 alike}) = \dfrac{6}{6^5}$

78) Values of $i = 2, 3, 4, \ldots, 12$

Total possible outcomes $= 36$

Must write down all outcomes and calculate as following:

$P(\text{sum is } i) = \dfrac{\#\text{ of outcomes that sum to } i}{36}$

8) a) 4 aces out of 52

$\therefore$ Required probability $= \dfrac{{}^{4}C_2}{{}^{52}C_2}$

b) 13 distinct values and 4 cards of each.

$\therefore$ Required probability $= \dfrac{13 \times {}^{4}C_2}{{}^{52}C_2}$

9) P(at least one 6 comes up) $= 1 - P(\text{No 6 comes up})$

$= 1 - \left(\dfrac{5}{6}\right)^4$ $\quad\because$ P(seeing one 6) $= 5/6$

10) If 2 dice are thrown, 1 out of 36 possible outcomes is a double six

$\therefore$ P(double six atleast once in n tosses) $= 1 - P(\text{No double 6})$

$= 1 - \left(\dfrac{35}{36}\right)^n$

Solving for $\dfrac{1}{2} \geq 1 - \left(\dfrac{36}{36}\right)^n$ we see,

Probability at $n = 24$ is less than 0.5 but crosses 0.5 at $\underline{\underline{n = 25}}$