HOMEWORK 4: Solutions
STAT 414 Spring 2015

1) X : Winnings when 2 balls are randomly chosen; where we have 8 W, 4 B, 2 O balls and win \$2 for a Black ball and lose \$1 for a White ball

- $P(X=-2) \equiv P(\text{choosing both white balls}) = \dfrac{{}^{8}C_2}{{}^{14}C_2}$
- $P(X=-1) \equiv P(\text{choosing one white and one orange ball}) = \dfrac{{}^{8}C_1\,{}^{2}C_1}{{}^{14}C_2}$
- $P(X=0) \equiv P(\text{choosing both orange balls}) = \dfrac{{}^{2}C_2}{{}^{14}C_2}$
- $P(X=1) \equiv P(\text{choosing one black and one white ball}) = \dfrac{{}^{8}C_1\,{}^{4}C_1}{{}^{14}C_2}$
- $P(X=2) \equiv P(\text{choosing one black and one orange ball}) = \dfrac{{}^{4}C_1\,{}^{2}C_1}{{}^{14}C_2}$
- $P(X=4) = P(\text{choosing both black balls}) = \dfrac{{}^{4}C_2}{{}^{14}C_2}$

2) Since we are rolling the dice twice, the total number of possible outcomes is 36.

a), b) The maximum value in the two rolls (and similarly minimum in part b) refers to the maximum among the two values observed on each of the two rolls.

A value will be considered the maximum when both dice have the same value or the the other die has a value lesser than this value.

A value is considered to be minimum when in the exactly opposite situation.

Note that outcomes of type (a, b) and (b, a) are considered as 2 possible out comes

Let X_1: Maximum value in 2 rolls of a die

$\therefore P(X_1 = 1) = 1/36$ $\quad P(X_1 = 2) = 3/36$

$P(X_1 = 3) = 5/36$ $\quad P(X_1 = 4) = 7/36$

$P(X_1 = 5) = 9/36$ $\quad P(X_1 = 6) = 11/36$

Let X_2: minimum value in 2 rolls of a die

$\therefore P(X_2 = 1) = 11/36$ $\quad P(X_2 = 2) = 9/36$

$P(X_2 = 3) = 7/36$ $\quad P(X_2 = 4) = 5/36$

$P(X_2 = 5) = 3/36$ $\quad P(X_2 = 6) = 1/36$

c) Same as Question 7: Homework 2

d) Let X_3: (Value of 1st roll) - (value of 2nd roll)

The maximum difference can be $|5|$ i.e. either when (6-1) occurs or (1-6) occurs.

$\therefore P(X = -5) = 1/36 = P(X = 5)$

$P(X = -4) = 2/36 = P(X = 4)$

$P(X = -3) = 3/36 = P(X = 3)$

$P(X = -2) = 4/36 = P(X = 2)$

$P(X = -1) = 5/36 = P(X = 1)$

$P(X = 0) = 6/36$

3) Let X: Total dollar value of all sales

Possible values and corresponding probabilities are:

- P(both appointments no sale) $= P(X = 0) = 0.7 \times 0.4 = 0.28$
- P(one of the sales is standard and other none) $= P(X = 500) = (0.3 \times 0.5 \times 0.4) + (0.7 \times 0.5 \times 0.6) = 0.27$
- P(Both sales standard or one deluxe) $= P(X = 1000) = (0.3 \times 0.5 \times 0.6 \times 0.5) + (0.3 \times 0.5 \times 0.4) + (0.7 \times 0.6 \times 0.5) = 0.315$
- P(One standard & one deluxe sale) $= P(X = 1500) = 2 \times (0.3 \times 0.5 \times 0.6 \times 0.5) = 0.09$
- P(Bothe deluxe sales) $= P(X = 2000) = (0.3 \times 0.5 \times 0.6 \times 0.5) = 0.045$

4) We are given,

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{4} & 0 \leq x < 1 \\ \frac{1}{2} + \frac{x-1}{4} & 1 \leq x < 2 \\ \frac{11}{12} & 2 \leq x < 3 \\ 1 & x \leq 3 \end{cases}$$

Now, $F(X = x) = P(X \leq x)$

Since, by the format in which the limits of this distribution function are indicated, we know that this is a discrete random variable.

$$\therefore P(X = i) = P(X \leq i) - P(X < i)$$
$$= F(i) - \lim_{n\to\infty} F(i - 1/n)$$

a) $P(X=1) = F(1) - \lim_{n\to\infty} F(1 - 1/n) = \frac{1}{2} + \frac{x-1}{4} - \frac{x}{4} = \frac{1}{2} + 0 - \frac{1}{4} = \frac{1}{4}$

$P(X=2) = F(2) - \lim_{n\to\infty} F(2 - 1/n) = \frac{11}{12} - \frac{1}{2} - \frac{x-1}{4} = \frac{11-6-3}{12} = \frac{1}{6}$

$P(X=3) = F(3) - \lim_{n\to\infty} F(3 - 1/n) = 1 - \frac{11}{12} = \frac{1}{12}$

★★ This question was written as $\frac{b}{4}$ and $\frac{1}{2} + \frac{b-1}{4}$ in the homework question paper.

This question, with bs as xs, was to be taken from the textbook.

b) $P(1/2 < X < 3/2) = P(0.5 < X < 1.5) = P(X = 1)$ Since that is the only possible value for X in the given interval

$= \frac{1}{4}$

5) $P(\text{red}) = 18/38$, $P(\text{not red}) = 20/38$

$+\$1$ per red and $-\$1$ per the other outcome

Let X: Winnings when one quits

a)∴ $P(X = -3) = P(\text{losing all 3 games}) = \frac{20}{38} \times \frac{20}{38} \times \frac{20}{38} = 0.15$

$P(X = -1) = P(\text{when first game lost and 1 of the next 2 won}) = \left(\frac{20}{38} \times \frac{18}{38} \times \frac{20}{38}\right) \times 2 = 0.26$

$P(X = 1) = P(\text{First game won OR first game lost and both others won}) = \frac{18}{38} + \left(\frac{20}{38} \times \frac{18}{38} \times \frac{18}{38}\right) = 0.59$

b) Although the probability of positive outcome is > 0.5, we must check if the expected value is positive. At the outset, it does not appear to be a definitive winning strategy. The initial probability of losing is higher than that of winning

c) $E(X) = (-3 \times 0.15) + (-1 \times 0.26) + (1 \times 0.59) = \underline{\underline{-0.12}}$

6) a) Calculation of E(X) gives more weight (than 1/4) for buses with students more than 37. This will also be weighted by the higher number. Therefore $E(X) > E(Y)$ since E(Y) gives equal weight to each bus.

b) $E(X) = 40 \times \frac{40}{148} + 33 \times \frac{33}{148} + 25 \times \frac{25}{148} + 50 \times \frac{50}{148} = 39.28$

$E(Y) = 40 \times \frac{1}{4} + 33 \times \frac{1}{4} + 25 \times \frac{1}{4} + 50 \times \frac{1}{4} = 148\left[\frac{1}{4}\right] = 37$

7)

a) There are 2 possibilities; either buy nothing or buy something in the first week and sell it later.

~~P(E~~

- E(money if nothing is ever purchased) $= \$1000$
- Let 'k' be the quantity bought in the first week

$\therefore$ E(buying 'k' in first week and selling next week money) $=$

$$\frac{1}{2}(1000-2k+k)+\frac{1}{2}(1000-2k+4k)$$

$$= 1000 + k/2$$

This can be maximized by increasing 'k' to its maximum ie. ~~5000~~ 500 (we have only \$1000)

$\therefore$ Best strategy is to buy 500 units in the first week and sell them all at the end.

b) E(content if bought this week) $= 500$

E(amount if bought next week) $= \frac{1}{2}(1000) + \frac{1}{2}\left(\frac{1000}{4}\right)$

$$= \frac{1000+250}{2}$$

$$= 625$$

$\therefore$ Any combination of these will only reduce the total quantity.

$\therefore$ Best strategy is to buy nothing in the first week and buy will all the money next week.

8) Let X: Number of heads on flipping 2 coins

a) $\therefore P(X=0) = P(\text{Both tails}) = 0.4 \times 0.3 = 0.12$

$P(X=1) = P(\text{one shows head}) = \begin{matrix}(0.6\times0.3)\\+(0.7\times0.4)\end{matrix} = 0.46$

$P(X=2) = P(\text{Both heads}) = 0.6 \times 0.7 = 0.42$

b) $E(X) = (0 \times 0.12) + (1 \times 0.46) + (2 \times 0.42)$

$E(X) = 1.3$

9) Say 'B' is the amount that the insurance company charges
Since they have to payout A with probability 'p'

$\therefore B - pA = 0.1A$

$\Rightarrow B = 0.1A + pA$

$\Rightarrow \boxed{B = (p+0.1)A}$

10) $E(X) = 1$, $var(X) = 5$

a) $E[(2+X)^2] = E(4 + 4X + X^2)$

$= 4 + 4E(X) + [var(X) + [E(X)]^2]$ $\quad var(X) = E(X^2) - [E(X)]^2$

$= 4 + 4 + 5 + 1$

$= 14$

b) $V(4 + 3X) = 0 + 3^2 V(X)$

$= 9 \times 5$

$= 45$