Math/Stat 414.4 Spring 2015 Exam 1 Name: ________

Instructions: Please read!

1. Do all work on this exam packet. It is okay to leave your answer unsimplified, as in $0.56\frac{15!}{6!9!}$ or $14e - 20$. This means that no calculator is needed.
2. Show all work for full credit! Small mistakes in arithmetic will not reduce credit if you show work; conversely, even a correct answer could get no credit without supporting work.
3. I will award partial credit where appropriate.
4. Each subproblem is worth 9 points. (You receive 1 point by including your name on the exam.)

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1. Drawer A contains 4 pennies and 5 dimes. Drawer B contains 6 pennies and 9 dimes. A drawer is selected at random, and a coin is selected at random from that drawer.

   (a) Find the probability of drawing a dime.

   We can use the law of total probability.

   $$P(dime) = P(dime|A)P(A) + P(dime|B)P(B)$$

   Filling in the appropriate values

   $$P(dime) = \frac{5}{9}\frac{1}{2} + \frac{9}{15}\frac{1}{2}$$

   (b) Suppose a dime is obtained. What is the probability it came from drawer B?

   Here, we apply Bayes rule

   $$P(B|dime) = \frac{P(dime|B)P(B)}{P(dime|A)P(A) + P(dime|B)P(B)} = \frac{\frac{9}{15}\frac{1}{2}}{\frac{5}{9}\frac{1}{2} + \frac{9}{15}\frac{1}{2}}$$

2. Let $P(A) = P(B) = 1/3$ and $P(A \cap B) = 1/10$ Find the following:

   (a) $P(B')$

   $$P(B') = 1 - P(B) = 1 - 1/3 = 2/3$$

   (b) $P(A' \cap B')$
   Apply DeMorgan's.

   $$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)) = 1 - 1/3 - 1/3 + 1/10 = 1/3 + 1/10$$

   (c) $P(A \cup B')$

   $$P(A \cup B') = P(A) + P(B') - P(A \cap B')$$

   and note that

   $$P(A) = P(A \cap B') + P(A \cap B)$$

   Putting these together

   $$P(A \cup B') = 1/3 + 2/3 - (1/3 - 1/10) = 2/3 + 1/10$$

3. A track star runs two races on a certain day. The probability that she wins the first race is 0.7. The probability that she wins the second race is 0.6. The probability that she wins both races is 0.5 Answer the following:

   (a) Find the probability that she wins at least one race.

   Define W_1 and W_2 as the events to win race one and two respectively.

   $$P(W_1 \cup W_2) = P(W_1) + P(W_2) - P(W_1 \cap W_2) = 0.6 + 0.7 - 0.5 = 0.8$$

   (b) Find the probability that she wins exactly one race.

   The probability can be expressed as the sum of the probabilities of the following mutually exclusive events.

   $$P(W_1 \cap W_2^c) + P(W_1^c \cap W_2) = P(W_1) - P(W_1 \cap W_2) + P(W_2) - P(W_1 \cap W_2) = 0.2 + 0.1$$

   (c) Find the probability that she wins neither race.

   This can be expressed by

   $$P(W_1^c \cap W_2^c) = P((W_1 \cup W_2)^c) = 1 - P(W_1 \cup W_2) = 1 - 0.8 = 0.2$$

   Note that we used DeMorgan's.

4. A club consists of 17 women and 13 men, and a committee of 5 members must be chosen.

   (a) How many committees are possible?

   There are

$$\binom{30}{5}$$

   ways to select a committee of five people.

   (b) How many committees are possible with two women and three men? Take the number of committees of two women from seventeen times the number of committees of three men from thirteen men

$$\binom{17}{2}\binom{13}{3}$$

   (c) Answer (b) if a particular man must be included.

   Since the particular man is on the committee, we now only need to count the number of committees with two women out of seventeen times the number of committees of two men out of twelve (remember the man that must be included is now no longer available for selection).

$$\binom{17}{2}\binom{12}{2}$$