Some extra problems from Chapter 8

no need to turn in

1. (8.1) Suppose that X is a random variable with mean and variance both equal to 20. What can be said about $P(0 < X < 40)$

2. (8.2) From past experience, a professor knows that the test score of a student taking her final examination is a random variable with mean 75.

   (a) Give an upper bound for the probability that a student's test score will exceed 85. Suppose, in addition, that the professor knows that the variance of a student's test score is equal to 25.

   (b) What can be said about the probability that a student will score between 65 and 85?

   (c) How many students would have to take the examination to ensure with probability at least 0.9 that the class average would be within 5 of 75? Do not use the central limit theorem.

   (d) (8.3) Repeat part (c) but using the central limit theorem.

3. (8.4) Let $X_1, ..., X_{20}$ be independent Poisson random variables with mean 1.

   (a) Use the Markov inequality to obtain a bound on

   $$P\left(\sum_{i=1}^{20} X_i > 15\right)$$

   (b) Use the central limit theorem to approximate

   $$P\left(\sum_{i=1}^{20} X_i > 15\right)$$

4. (8.6) A die is continually rolled until the sum of all rolls exceeds 300. Approximate the probability that at least 80 rolls are necessary.

5. (8.7/8.8) A person has 100 light bulbs whose lifetimes are independent exponentials with a mean of 5 hours.

   (a) If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours.

   (b) Suppose that it takes a random time, uniformly distributed over $(0, 0.5)$ to replace a failed bulb. Approximate the probability that all bulbs have failed by time 550.

6. An insurance company has 10,000 automobile policy holders. The expected yearly claim per policy holder is $240, with a standard deviation of $800. Approximate the probability that the total yearly claim exceeds $2.7 million.