MATH/STAT 414 HW 10

due April 23, 2015

1. (7.4) If X and Y have joint density function

$$f(x, y) = 1/y \quad I_{\{0<x<y<1\}}$$

   find

   (a) E[XY]

   (b) E[X]

   (c) E[Y]

2. (7.6) A fair die is rolled 10 times. Calculate the expected value and the variance of the sum of 10 rolls.

3. (7.16) Let Z be a standard normal random variable, and, for a fixed x, set

$$X = Z \ if \ Z > x$$

   and zero if $Z \leq x$. Show that

$$E[X] = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

   Also, find the variance of X.

4. (7.30) If X and Y are independent and identically distributed with mean μ and variance σ^2, find

$$E[(X - Y)^2]$$

5. (7.33) If $E[X] = 1$ and $Var[X] = 5$, find

   (a) $E[(2 + X)^2]$

(b) $Var(4+3X)$

6. (7.38) The random variables X and Y have a joint density function given by

$$f(x,y) = 2e^{-2x}/xI_{\{0<y<x<\infty\}}$$

Computer $Cov(X,Y)$.

7. (7.39) Let $X_1, X_2, ...$ be independent with common mean μ and common variance σ^2. Set $Y_n = X_n + X_{n+1} + X_{n+2}$. For $j \geq 0$, find $Cov(Y_n, Y_{n+j})$.

8. (7.50) The joint density of X and Y is given by

$$f(x,y) = \frac{e^{-x/y}e^{-y}}{y} I_{\{x>0,y>0\}}$$

Compute $E[X^2|Y=y]$.

9. (7.65) The number of winter storms in a good year is a Poisson random variable with mean 3, whereas the number in a bad year is a Poisson random variable with mean 5. If next year will be a good year with probability 0.4 or a bad year with probability 0.6, find the expected value and variance of the number of storms that will occur.

10. (7.68) The number of accidents that a person has in a given year is a Poisson random variable with mean λ. However suppose that the value of λ changes from person to person, being equal to 2 for 60 percent of the population and 3 for 40 percent. If a person is chosen at random, what is the probability that he will have (a) 0 accidents and (b) exactly 3 accidents in a certain year? What is the conditional probability that he will have 3 accidents in a given year, given that he had no accidents the proceeding year?

11. Suppose that X_1 and X_2 are independent exponentially distributed random variables. Let $Y = X_1 - X_2$. Find the moment generating function of Y.

12. Find the moment generating function of a random variable with density

$$f(x) = \left(\frac{1}{2}x + \frac{1}{2}\right) I_{\{-1<x<1\}}$$