MATH/STAT 414 HW 3

due February 5, 2015

1. (3.4) What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is i (where i could be 2, 3,...,12).

2. (3.6) Consider an urn containing 12 balls, of which 8 are white. A sample of size 4 is to be drawn with replacement (without replacement). What is the conditional probability (in each case) that the first and third balls drawn will be white given that the sample drawn contains exactly 3 white balls?

3. (3.10) Three cards are randomly drawn, without replacement, from an ordinary deck of 52 playing cards. Compute the conditional probability that the first card selected is a spade given that the second and third cards are spades.

4. (3.13) Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that the ith has exactly one ace. Determine $P = P(E_1 \cap E_2 \cap E_3 \cap E_4)$ using the multiplication rule.

5. (3.16) Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed the baby survives 96 percent of the time. If a randomly chosen pregnant woman does not have a C sections, what is the probability that the baby survives?

6. (3.40) Consider a sample of size 3 drawn in the following manner: We start with an urn containing 5 white and 7 red balls. At each stage, a ball is drawn and its color is noted. The ball is then returned to the urn, along with an additional ball of the same color. Find the probability that the sample will contain

(a) 0 white balls
(b) 1 white ball
(c) 2 white balls
(d) 3 white balls

7. (3.45) Suppose we have 10 coins such that if the ith coin is flipped, heads will appear with probability $i/10, i = 1, 2, ..., 10$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fifth coin?

8. (3.50)Suppose that an insurance company classifies people into one of three classes: good risks, average risks, and bad risks. The company's records indicate that the probabilities that good-, average-, and bad-rick persons will be involved in an accident over a 1-year span are, respectively, 0.05, 0.15, and 0.30. If 20 percent of the population is a good risk, 50 percent an average risk, and 30 percent a bad risk, what proportion of people have accidents in a fixed year? If policyholder A had no accidents in 2012, what is the probability that he or she is a good risk? is an average risk?

9. (3.55) In a class, there are 4 first year boys, 6 first year girls, and 6 sophomore boys. How many sophomore girls must be present if sex and class are to be independent when a student is selected at random.

10. (3.58) Suppose that we want to generate the outcome of the flip of a fair coin, but that all we have at our disposal is a biased coin that lands on heads with some unknown probability p that need not be equal to 1/2. Consider the following procedure for accomplishing our task:

    1 Flip the coin.
    2 Flip the coin again.
    3 If both flips land on heads or both land on tails, return to step 1.
    4 Let the result of the last flip be the result of the experiment.

    (a) Show that the result is equally likely to be either heads or tails.
    (b) Could we use a simpler procedure that continues to flip the coin until the last two flips are different and then have the result be the outcome of the final flip?