MATH/STAT 414 HW 4

due February 19, 2015

1. (4.1) Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let X denote our winnings. What are the possible values of X, and what are the probabilities associated with each value?

2. (4.7 & 4.8) Suppose that a die is rolled twice. What are the possible values that the following random variables can take on and calculate the probabilities associate with each of those values.

   (a) the maximum value to appear in the two rolls

   (b) the minimum value to appear in the two rolls

   (c) the sum of the two rolls

   (d) the value of the first roll minus the value of the second roll

3. (4.13) A salesman has scheduled two appointments to sell encyclopedias. (Feel free to ask your parents or other older people that you may know what encyclopedias were and why they were once important.) His first appointment will lead to a sale with probability 0.3, and his second will lead independently to a sale with probability 0.6. Any sale made is equally likely to be either for the deluxe model, which costs \$1000, or the standard model, which costs \$500. Determine the probability mass function of X, the total dollar value of all sales.

4. (4.17) Suppose that the distribution function of X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{b}{4} & 0 \leq x < 1 \\ \frac{1}{2} + \frac{b-1}{4} & 1 \leq x < 2 \\ \frac{11}{12} & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases}$$

(a) Find $P(X = i), i = 1, 2, 3$.

(b) Find $P(1/2 < X < 3/2)$.

5. (4.20) A gambling book recommends the following "winning strategy" for the game of roulette: Bet \$1 on red. If red appears (which as probability 18/38), then take the \$1 profit and quit. If red does not appear and you lose this bet (which has probability 20/38 of ocurring, make additional \$1 bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote your winnings when you quit.

   (a) Find $P(X > 0)$.

   (b) Are you convinced that the strategy is indeed a "winning strategy"? Explain your answer.

   (c) Find $E[X]$.

6. (4.21) Four buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students who were on the bus carrying the randomly selected student. One of the bus drivers is also randomly selected. Let Y denote the number of students on her bus.

   (a) Which of $E[X]$ and $E[Y]$ do you think is larger? Why?

   (b) Compute $E[X]$ and $E[Y]$.

7. (4.23) You have \$1000, and a certain commodity presently sells for \$2 per ounce. Suppose that after one week the commodity will sell for either \$1 or \$4 an ounce, with these two possibilities being equally likely.

   (a) If your objective is to maximize the expected amount of money that you possess at the end of the week, what strategy should you employ?

   (b) If your objective is to maximize the expected amount of the commodity that you posses at the end of the week, what strategy should you employ?

8. (4.25) Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.7. Assume that the results of the flips are independent, and let X equal the total number of heads that result.

(a) Find $P(X = 1)$

(b) Determine $E[X]$.

9. (4.27) An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p, what should it charge the customer in order that its expected profit will be 10 percent of A?

10. If $E[X] = 1$ and $Var[X] = 5$, find

    (a) $E[(2 + X)^2]$

    (b) $Var[4 + 3X]$