

# MATH/STAT 414 HW 6

due March 5, 2015

1. (4.71) Consider a roulette wheel with 38 numbers: 1 through 36, 0 and double 0. If Smith always bets that the outcome will be one of the numbers from 1 to 12, what is the probability that
  - (a) Smith will lose his first five bets
  - (b) his first win will be on bet 4.
2. (4.75) A fair coin is continually flipped until heads appears for the 10th time. Let  $X$  denote the number of tails that occur. Calculate the probability mass function of  $X$ .
3. (4.78) An urn contains 4 white and 4 black balls. We randomly choose 4 balls. If 2 are white and 2 are black, we stop. If not, we replace the balls in the urn and again select 4 balls. This continues until exactly 2 of the 4 balls are white. What is the probability that we make exactly  $n$  selections.
4. (4.79) Suppose that a batch of 100 items contains 6 defects and 94 that are not defective. If  $X$  is the number of defects in a randomly drawn sample of 10 items from the batch, find
  - (a)  $P(X = 0)$
  - (b)  $P(X > 2)$
5. (5.2) A system consisting of one original unit plus a spare can function for a random amount of time  $X$ . If the density of  $X$  is given (in units of months) by

$$f(x) = Cxe^{-x/2}I_{\{x>0\}}$$

what is the probability that the system functions for at least 5 months?

6. (5.4) The probability density function of  $X$ , the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \frac{10}{x^2} I_{\{x > 10\}}$$

- (a) Find  $P(X > 20)$ .
  - (b) What is the cumulative distribution function of  $X$ ?
  - (c) What is the probability that of 6 such types of devices, at least 3 will function for at least 15 hours? What assumptions are you making?
  - (d) Find  $E[X]$ .
7. (5.5) A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable with probability density function

$$f(x) = 5(1 - x)^4 I_{\{0 < x < 1\}}$$

What must be the capacity of the tank be so that the probability of the supply being exhausted in a given week is 0.01.

8. (5.7) The density of  $X$  is given by

$$f(x) = (a + bx^2) I_{\{0 < x < 1\}}$$

If  $E[X] = 3/5$ , find  $a$  and  $b$ .