MATH/STAT 414 HW 7

due March 19, 2015

1. (5.8) The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} I_{\{x>0\}}$$

   Compute the expected lifetime of such a tube.

2. (5.11) A point is chosen at random on a line segment of length L. Interpret this statement, and find the probability that the ration of the shorter to the longer segment is less than $1/4$.

3. (5.13) You arrive at a bust stop at 10 am, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

   (a) What is the probability that you will have to wait longer than 10 minutes?

   (b) If, at 10:15, the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

4. (5.16) The annual rainfall in inches in a certain region is normally distributed with $\mu = 40$ and $\sigma = 4$. What is the probability that starting with this year, it will take more than 10 years before a year occurs having a rainfall of more than 50 inches? What assumptions are you making?

5. (5.18) Suppose that X is a normal random variable with mean 5. If $P(X > 9) = 0.2$, approximately what is $Var(X)$?

6. (5.20) If 65 percent of the population of a large community is in favor of a proposed rise in school taxes, approximate the probability that a random sample of 100 people will contain

(a) at least 50 who are in favor of the proposition:

(b) between 60 and 70 inclusive who are in favor;

(c) fewer than 75 in favor.

7. (5.26) Two types of coins are produced at a factory: a fair coin and a biased one that comes up heads 55 percent of the time. We have one of these coins but do not know whether it has a fair coin or a biased one. In order to ascertain which type of coin we have, we shall perform the following statistical test: We shall toss the coin 1,000 times. If the coin lands on heads 525 or more times, then we shall conclude that it is a biased coin, whereas if it lands on heads fewer than 525 times, then we shall conclude that it is a fair coin. If the coin is actually fair, what is the probability that we shall reach a false conclusion? What would it be if the coin were biased?

8. (5.27) In 10,000 independent tosses of a coin, the coin landed on heads 5800 times. Is it reasonable to assume that the coin is not fair? Explain.

9. (5.31)

   (a) A fire station is to be located along a road of length A (with $A < \infty$). If fires occur at points uniformly chosen on $(0, A)$, where should the station be located so as to minimize the expected distance from the fire? That is, choose a so as to minimize

   $$E[|X - a|]$$

   where X is uniformly distributed over $(0, A)$.

   (b) Now, suppose that the road is of infinite length–stretching form point 0 outward to ∞. If the distance of a fire from point 0 is exponentially distributed with rate λ, where should the fire station now be located? That is, we want to minimize $E[|X - a|]$, where X is now exponentially distributed with rate λ.

10. (5.33) The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a used radio that is 3 years old, what is the probability that it will be working after an additional 8 years after he buys it?

11. (5.37) If X is uniformly distributed over $(-1, 1)$, find

(a) $P(|X| > 1/2)$

(b) the density function of the random variable $|X|$.

12. (5.39) If X is an exponential random variable with parameter $\lambda = 1$, compute the probability density function of the random variable$y$ defined by $Y = \log(X)$.