MATH/STAT 414 HW 8

due April 9, 2015

1. (6.2) Suppose that 3 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let X_i equal 1 if the ith ball is selected is white, and let it equal 0 otherwise. Give the joint probability mass function of

   (a) X_1, X_2

   (b) X_1, X_2, X_3

2. (6.6) A bin of 5 transistors is known to contain 2 that are defective. The transistors are to be tested, one at a time, until the defective ones are identified. Denote by N_1, the number of tests made until the first defective is identified and by N_2 the number of additional tests until the second defective is identified. Find the joint probability mass function of N_1 and N_2.

3. (6.9) The joint probability density function of X and Y is given by

$$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) \quad 0 < x < 1 \quad 0 < y < 2$$

   (a) Verify that this is indeed a joint density function.

   (b) Compute the density function of X.

   (c) Find $P(X > Y)$.

   (d) Find $P(Y > 1/2 | X < 1/2)$.

   (e) Find $E[X]$.

   (f) Find $E[Y]$.

4. (6.14) An ambulance travels back and forth at a constant speed along a road of length L. At a certain moment of time, an accident occurs at a point uniformly distributed on the road. Assuming that the

ambulance's location at the moment of the accident is also uniformly distributed, and assuming independence of the variables, compute the distribution of the distance of the ambulance from the accident.

5. (6.15) The random vector (X, Y) is said to be uniformly distributed over a region R in the plane if, for some constant c, its joint density is

$$f(x, y) = cI_{\{(x,y)\in R\}}$$

   (a) Show that $1/c$ =area of region R.
   (b) Suppose that (X, Y) is uniformly distributed over the square centered at $(0, 0)$ and with sides of length 2. Show that X and Y are independent with each being distributed uniformly over $(-1, 1)$.
   (c) What is the probability that (X, Y) lies in the circle of radius 1 centered at the origin? That is find $P(X^2 + Y^2 \leq 1)$.

6. (6.17) Three points X_1, X_2, X_3 are selected at random (and independently) on a line L. What is the probability that X_2 lies between X_1 and X_3?

7. (6.19) Show that $f(x, y) = 1/x, 0 < y < x < 1$ is a joint density function. Assuming that f is the joint density function of X, Y find
   (a) the marginal density of Y
   (b) the marginal density of X
   (c) $E[X]$
   (d) $E[Y]$.

8. (6.22) The joint density function of X and Y is

$$f(x, y) = x + y \quad 0 < x < 1,\ 0 < y < 1$$

   (a) Are X and Y independent?
   (b) Find the density function of X.
   (c) Find $P(X + Y < 1)$.

9. (6.27) If X_1 and X_2 are independent exponential distributions with respective parameters λ_1 and λ_2, find the distribution of $Z = X_1/X_2$. Also, compute $P(X_1 < X_2)$.