MATH/STAT 414 HW 9

due April 16, 2015

1. (6.38) Choose a number X at random from the set of number $\{1,2,3,4,5\}$. Now choose a number at random from the subset no larger than X, that is from $\{1,2,...,X\}$. Call this number Y.
   (a) Find the joint mass function of X and Y.
   (b) Find the conditional mass function of X given that $Y = i$. Do this for $i = 1,2,3,4,5$.
   (c) Are X and Y independent? Why?

2. (6.39) Two dice are rolled. Let X and Y denote, respectively, the largest and smallest values obtained. Compute the conditional mass function of Y given $X = i$, for $i = 1,2,...,6$. Are X and Y independent? Why?

3. (6.40) The joint probability mass function of X and Y is given by

$$p(1,1) = 1/8 \quad p(1,2) = 1/4 \quad p(2,1) = 1/8 \quad p(2,2) = 1/2$$

   (a) Compute the conditional probability mass function of X given $Y = i, i = 1,2$.
   (b) Are X and Y independent?
   (c) Compute $P(XY \leq 3)$, $P(X+Y > 2)$, $P(X/Y > 1)$.

4. (6.41) The joint density function of X and Y is given by

$$f(x,y) = xe^{-x(y+1)} \quad x > 0 \quad y > 0$$

   (a) Find the conditional density of X, given $Y = y$, and that of Y, given $X = x$.
   (b) Find the density function of $Z = XY$.

5. (6.42) The joint density of X and Y is

$$f(x, y) = c(x^2 - y^2)e^{-x} \; 0 < x < \infty, \; -x < y < x$$

Find the conditional distribution of Y given $X = x$.

6. (6.43) An insurance company supposes that each person has an accident parameter and that the yearly number of accidents of someone whose accident parameter is λ is Poisson distributed with mean λ. They also suppose that the parameter value of a newly insured person can be assumed to be the value of a gamma random variable with parameters s and α. If a newly insure person has n accidents in her first year, find the conditional density of her accident parameter. Also, determine the expected number of accidents that she will have in the following year.