

Instructions: Please read!

1. Do all work on this exam packet. It is okay to leave your answer unsimplified, as in $0.56 \frac{15!}{6!9!}$ or $14e - 20$. This means that no calculator is needed.
 2. Show all work for full credit! Small mistakes in arithmetic will not reduce credit if you show your work; conversely, even a correct answer could get no credit without supporting work.
 3. I will award partial credit where appropriate.
-

1. (10 points) Derive the moment generating function for X , a binomial random variable with parameters n and $0 < p < 1$. Note that your final answer should be in closed form (i.e. with no summation symbol (Σ)).

2. (10 points) The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/7$. If Jones buys a previously owned radio that has been in use for eight years, what is the probability that it will be working after an additional 3 years?

3. (10 points) Let X have the density

$$f(x) = \frac{2}{3}xI_{\{-1 < x < 2\}}$$

Find the density for X^2 .

4. (10 points) Suppose we have 10 coins such that if the i th coin is flipped, heads will appear with probability $i/10$, $i = 1, 2, \dots, 10$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fifth coin? (Fact: the sum of the first n strictly positive integers is $\frac{n(n+1)}{2}$)

5. (10 points) If X and Y are independent and identically distributed with mean μ and variance σ^2 , find

$$E[(X - Y)^2]$$

6. The lifetimes of interactive computer chips produced by a certain semiconductor manufacturer are exponentially distributed with rate $\lambda = \frac{1}{1,400,000}$. The chips come in batches of 100 chips.
- (a) (5 points) What is the approximate probability that a batch will contain at least 20 chips whose lifetimes are less than 1,800,000?

- (b) (10 points) Suppose all of the chips in a batch go into service at the same time, find the probability density function for the time until the first chip in the batch fails.

7. The joint probability density function of X and Y is given by

$$f(x, y) = \frac{6}{7} \left(x^2 + \frac{xy}{2} \right) I_{\{0 < x < 1, 0 < y < 2\}}$$

(Note that this is the last problem on the exam. The rest of the subparts to follow are all concerning this joint density.)

(a) (5 points) Verify that this is indeed a joint density function.

(b) (5 points) Compute the density function of X .

(c) (5 points) Find $P(X > Y)$.

(d) (7.5 points) Find

$$P\left(Y > \frac{1}{2} \mid X < \frac{1}{2}\right)$$

(e) (5 points) Compute the density function for Y given $X = x$.

(f) (7.5 points) Find $Var[Y]$.