

STAT 440, Homework 1

due 1.22.2015

1. Let X have density $12(1-x)x^2$ for $0 < x < 1$ and zero otherwise.
 - (a) Find the mean and variance for this random variable.
 - (b) Find $E[X^3]$.
 - (c) Find the $P(0 < X < 0.5)$.
2. Let U_1, U_2, \dots be a sequence of uniform random variables from zero to one. Find a_n and b_n , so that

$$\frac{\sum_{i=1}^n U_i + a_n}{b_n}$$

converges to a standard normal in distribution. What does it mean to converge in distribution? This is an application of what theorem? Now, select a different sequences a_n and b_n so that

$$\frac{\sum_{i=1}^n U_i + a_n}{b_n}$$

converges to the number zero. This is an application of what theorem?

3. Suppose that X_1, \dots, X_n are iid random variables with common density

$$f_\theta(x) = \theta^2 x e^{-\theta x} \quad x > 0 \quad \theta > 0$$

- (a) Find the MLE for θ
 - (b) Calculate the Fisher Information for X_1, \dots, X_n and use it to find an approximate 95% confidence interval for θ .
4. Do the following R activities:
 - (a) Give an R command to sum all the odd integers from 1 to 199.

- (b) Write an R function which takes a vector as an argument, calculates the geometric mean, and then returns that value.
- (c) Generate one hundred standard normal random variables (using the `rnorm` command), then use the `hist` command to create a histogram. Make sure to create a relative frequency histogram.
- (d) Write an R function which takes a vector as an argument and calculates the sample third moment, then returns that value.