STAT 440, Homework 3

due 2.19.2015

1 Turn-in on paper

1. Write down an accept-reject algorithm to generate a $Beta(a, b)$ density. Be sure to verify the appropriate conditions.

2. Generate a random sample of 1000 observations from a normal location mixture. (Be sure to read section 3.5.) The components of the mixture should be $N(0, 1)$ and $N(3, 1)$. You may either use `rnorm()` or the function you wrote in the last homework to generate normals.

   (a) Graph the histogram of the mixture with the density superimposed when the probability of the first component (i.e. the standard normal) is 0.75. (Turn in the plot)

   (b) Repeat with different mixing probabilities and speculate about which values of the mixing probability will yield a bimodal density. Provide evidence through plots.

3. Use the Choleski method to generate 300 observations from a multivariate normal. (The function is essentially given in the chapter. You may either use `rnorm()` or the function you wrote in the last homework to generate normals.) Input this into R, but pay attention to the implementation. The components should all have variance one and zero mean. The correlation between components one and two should be 0.5, the correlation between components two and three should be -0.25, and the correlation between components one and three should be 0.25. Use the `pairs` command to verify the implementation. Provide graphical support that your simulation is being drawn for the correct distribution.

4. Compute a Monte Carlo estimate of

$$\int_0^{\pi/3} sin(t)dt$$

and compare your estimate to the exact value of the integral.

2 Turn-in through Angel

1. Write a function to implement the accept-reject method to generate a $Beta(a, b)$ density. Use the following naming for your function

```
betasim(n,a,b)
```

Your function will be tested by comparing in to a $Beta(3, 2)$ simulated from the built in function.

2. Write a function that will generate a Monte Carlo estimate of the normal cdf, $\Phi(x)$, for $x > 0$. *Have your function return the 95% confidence interval for the estimate.* Use the following naming scheme.

```
mcphi(x,m)
```

where `m` is the number of simulations used to generate the Monte Carlo estimate.