Math/Stat 416.2 Quiz 2 Name: ____________________

Instructions: Please read!

1. Do all work on this exam packet. It is okay to leave your answer unsimplified, as in $0.56\frac{15!}{6!9!}$ or $14e - 20$. This means that no calculator is needed.
2. Show all work for full credit! Small mistakes in arithmetic will not reduce credit if you show work; conversely, even a correct answer could get no credit without supporting work.
3. I will award partial credit where appropriate.
4. You may use all the facts from the probability review as given facts UNLESS you are explicitly asked to derive something.

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1. (25 pts) Suppose that Y has a uniform distribution on the interval from 0 to 1. The conditional distribution of random variable X given $Y = y$ has a binomial distribution with parameters $n = 10$ and $p = y$. Find the variance of X.

   solution: $Var(X) = Var[E[X|Y]] + E[Var[X|Y]]$. From what we know about the binomial distribution, i.e. its mean and variance, we have

   $$Var(X) = Var[nY] + E[nY(1-Y)] = n^2 Var[Y] + nE[Y] - nE[Y^2]$$

   Using what we know from the uniform distribution

   $$Var(X) = (10)^2/12 + 10/2 - 10/3$$

2. (25 pts) A taxicab driver moves between the airport A and two hotels, B and C, according to the following rules. If he is at the airport, then he will go to one of the two hotels next with equal probability. If he is at one of the hotels, then he will go to the airport with probability $3/4$ and to the other hotel with probability $1/4$.

   (a) Write down the probability transition matrix for the chain describing the taxi's location. solution:

$$P = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{pmatrix}$$

   (b) Suppose the driver starts at the Hotel B at time 0. Find the probability for each of his possible three locations at time 2.

   solution: To find the two step transition probabilities, we need to square the probability transition matrix.

$$P^2 = \begin{pmatrix} 6/8 & 1/8 & 1/8 \\ 3/16 & 7/16 & 3/8 \\ 3/16 & 3/8 & 7/16 \end{pmatrix}$$

   So, the answer is the last row. $P(X_2 = 1|X_0 = 2) = 3/16, P(X_2 = 2|X_0 = 2) = 7/16, P(X_2 = 3|X_0 = 2) = 3/8$.

3. (25 pts) Data indicate that the number of traffic accidents in Berkeley on a rainy day is a Poisson random variable with mean 9, whereas on a dry day it is a Poisson random variable with mean 3. Define the number of accidents in Berkeley tomorrow to be random variable X. The probability that it will rain tomorrow is 0.6.

   (a) Find $E[X]$ solution: $E[X] = E[X|rain]P(rain)+E[X|no\,rain]P(no\,rain)$. So, $E[X] = 9(0.6) + 3(0.4) = 6.8$

   (b) Find $P[X = 0]$ solution: $P[X = 0] = P[X = 0|rain]P(rain) + P[X = 0|no\ rain]P(no\ rain) = 0.6e^{-9} + 0.4e^{-4}$

4. (25 pts) A coin, having probability p of landing heads, is continually flipped until at least one head and one tail have been seen. Find the expected number of flips needed. solution: Condition on the first flip and call the number of trials to obtain at least one head and one tail X.

$$E[X] = E[X|h]P(h) + E[X|t]P(t) = E[X|h]p + E[X|t](1-p)$$

Now, for $E[X|h]$ we are now waiting for the next tail, so the expectation is one more than the expected value of a geometric distribution with parameter $1-p$, i.e. $1 + 1/(1-p)$. Similarly, $E[X|t]$ is equal to one more than the expected value of a geometric distribution with parameter p, i.e. $1 + 1/p$. So,

$$E[X] = (1 + 1/(1-p))p + (1 + 1/p)(1-p) = 1 + \frac{p^2 + (1-p)^2}{p(1-p)}$$